Continuation Options and Returns-Earnings Convexity[†]

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April 2012

NYU Stern Department of Accounting Working Paper ACC-11-01

Abstract

We hypothesize and provide evidence that convexity in the returns-earnings relation results in significant part from firms' real continuation options, i.e., their discretionary ability to continue operations, to make new investments, and to raise capital when financing deficits arise. We estimate convexity using spline regressions for sequential partitions of the sample based on proxies for general optionality, liquidation likelihood, and continuation optionality. We develop four measures of convexity that capture the positive implications of earnings uncertainty from the perspective of equityholders. We predict and find that all four measures are significantly positively (negatively) associated with firms' future equity (debt) financings. These results suggest that our convexity measures have general usefulness for research in accounting and finance, which typically focuses on the negative implications of earnings uncertainty.

Keywords: continuation; real options; returns-earnings; convexity; financing; lifecycle; volatility

JEL Classification: G32; M41

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† We thank participants of the accounting research workshops at Southern Methodist University, the University of Pennsylvania and the University of Virginia, the NYU Stern Accounting Ph.D. empirical research seminar, and the JAAF/KPMG Foundation Conference (in particular, Masako Darrough) and Bob Jennings for helpful comments and suggestions.

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[1] Introduction

The empirical accounting literature (e.g., Beaver, Clarke and Wright 1979, Freeman and Tse 1992, and Hayn 1995) shows that the relation between returns and earnings is convex over the range from the most negative earnings ("losses") to moderately positive earnings ("gains").[1] The theoretical and empirical accounting literatures explain this convexity in terms of firms' real option[2] to abandon or to adapt the use of assets in place (Hayn 1995), the limited liability of common stock (Fischer and Verrecchia 1997), the frequent reversal of losses over time (Joos and Plesko 2005), and the existence of unrecognized economic assets and financing opportunities for loss firms (Darrough and Ye 2007). While these explanations are distinct, they all exhibit the underlying theme that the responsiveness of returns to losses is attenuated relative to the (normal) responsiveness of returns to gains. That is, incremental losses do not indicate downside realizations of risk to the same extent as incremental gains indicate upside realizations of risk. For this reason, we refer to these prior papers collectively as explaining returns-earnings convexity ("convexity") in terms of "left-tail options."

We hypothesize and provide evidence that convexity also results in significant part from firms' real continuation options, i.e., their discretionary ability to continue operations, to make new investments, and to raise capital when financing deficits arise. Darrough and Ye (2007), the most related prior paper, show that loss firms' raising of capital, one type of continuation option, attenuates the responsiveness of returns to losses. Complementing and extending their finding, we provide evidence that continuation options associated with firms' investments in and financing of real options accentuate the responsiveness of returns to gains over a potentially wide range from the middle to right side (but not necessarily the tail) of the earnings distribution.

[1] Returns may exhibit lower sensitivity to extreme gains because they indicate sufficiently high probabilities of continuation or are sufficiently transitory (Freeman and Tse 1992).
[2] Real options involve uncertain and at least partly irreversible (dis)investments in non-financial economic assets.

Over this range, incremental gains are associated with an increased probability of or benefits to continuation, and thus more strongly indicate upside realizations of risk.

We refer to our paper as explaining convexity in terms of "right-tail options" (shorthand for the more accurate but less wieldy term "right-of-left-tail options") that are distinct from the left-tail options explored by prior research.[3] Collectively, left- and right-tail options pull the returns-earnings relation upward from the left side and from the middle-to-right side of the distribution of earnings, yielding convexity. Stronger options enhance these effects.

We identify the presence of continuation options relying on the theoretical and empirical literature in financial economics that shows that investments in real options—in particular, research and development ("R&D")—typically are financed in stages using equity (e.g., Myers 1977, Trigeorgis 1993, Hall and Lerner 2010, and Bergemann, Hege, and Peng 2011). Stage financing inherently yields continuation options for firms investing in real options.

We develop a simple approach to estimate overall convexity as well as the distinct contributions of left- and right-tail continuations options to overall convexity. We regress annual stock returns on annual deflated earnings per share ("EPS") using spline regressions with a single

[3] We do not mean to suggest that we are the first to contemplate the possibility of right-tailed continuation or other options in accounting. For example, in a discussion of what makes accounting information useful, Lambert (2010) provides this lucid explanation of the importance of right-tailed continuation options:

> ...to be most valuable, information should be the most precise where it impacts decisions the most...this is on the good news (the profit) side, not the bad news side...To see this, consider first the decision to invest in new projects. If information suggests a project is profitable, then more detailed information is valuable because, in general, the more profitable the investment, the more you'd invest in it. On the other hand, if information suggests that a project is unprofitable, there is relatively little value to more precise information. This is because your decision will be the same—invest nothing—regardless of whether the information indicates the project is very bad as opposed to just bad...Therefore, decisions are more sensitive to information in the upper tail for claimholders as a whole...

In addition, a longstanding literature dating back at least to Miller and Modigliani (1961, 1966) discusses the implications of growth opportunities or options for the multiples on earnings, usually in levels valuation models. While similar in some respects, the growth considered in this literature is most naturally viewed as later-lifecycle-stage options descriptive of more mature firms than the research and development investment and equity financing options that are our focus. Using the language in the text, these growth options are further to the right of of our right-of-left-tail continuation options. Notable recent accounting papers in this literature include Zhang (2000), Chen and Zhang (2007), and Hao, Jin, and Zhang (2011).

knot. We show these regressions fit the data remarkably well, although we emphasize that our goal is not to maximize goodness of fit. Using the output of these regressions, we propose and employ four measures of convexity. Our first two measures capture overall convexity arising from both left- and right-tail options. The first measure is the difference between the slope coefficients on deflated EPS to the left versus the right of the spline knot, consistent with the emphasis on the differential slope coefficients for losses and gains in the prior literature. The second measure is the vertical distance between the OLS fitted line and the spline knot. This measure incorporates the difference between the slope coefficients as well as the dispersion of deflated EPS. For this reason, it more fully captures the limitations of estimating the returns-earnings relation without incorporating convexity. The third and fourth measures capture the contribution to overall convexity arising individually from left- and right-tail options, respectively. The third measure is the negative of the slope coefficient on deflated EPS to the left of the spline knot.[4] The fourth measure is the slope coefficient on deflated EPS to the right of the spline knot. For simplicity, we refer to these measures as left- and right-tail convexity, respectively, although they are properly viewed as contributions to overall convexity, because we do not allow for convexity within each segment of the spline regression.

We predict that the four convexity measures vary across subsamples formed by sequentially partitioning firms each year based first on two control variables related to the optionality of equity in general and the left-tail options examined in prior research and then on two test variables related to continuation options. This sequence highlights the incremental importance of continuation options, particularly right-tail options.

The first control variable is stock return volatility ("volatility"). Volatility must exist for

[4] We take the negative of the slope coefficient for the segment to the left of the spline knot because a lower (e.g., more negative) coefficient indicates greater convexity.

both left- and right-tail options to be valuable. We predict and find that all four measures of convexity rise with volatility. Because of the critical importance of volatility, our subsequent analysis focuses on high volatility observations. The second control variable is stock price. Lower stock prices indicate firms that are more likely to liquidate assets in place and for which stock behaves more like an at-the-money purchased call option on those assets, i.e., firms with stronger left-tail options. We refer to the inverse of stock price as liquidation likelihood. We predict and find that the overall and left-tail measures of convexity are higher for above-median liquidation likelihood.

The first test variable is R&D intensity, which we refer to as investment real optionality. Joos and Plesko (2005) and Darrough and Ye (2007) show that R&D attenuates the responsiveness of returns to losses. Based on the aforementioned financial economics literature, we predict that investment real optionality also accentuates the responsiveness of returns to gains. Consistent with this prediction, we find that all four measures of convexity are higher for high investment real optionality firms, regardless of liquidation likelihood level.

The second test variable is positive net issuances of equity, which we refer to as market continuation. Darrough and Ye (2007) show that both equity and debt financing attenuates the responsiveness of returns to losses. As discussed above, equity is the most natural form of external financing for firms investing in real options. This is particularly true for firms whose economic assets are primarily intangible and thus serve poorly as collateral (e.g., Carpenter and Petersen 2002 and Brown, Fazzari, and Petersen 2009). As firms may issue equity for many reasons, we focus our analysis of market continuation on firms with high investment real optionality. We predict and find that all four measures of convexity are higher for firms with market continuation.

We show that incorporating average convexity (i.e., estimating a spline rather than OLS regression) in the overall sample raises the explanatory power of earnings for returns from approximately 5.1% to 8.7%, consistent with the increase in explanatory power documented by Freeman and Tse (1992).[5] Allowing convexity to vary across the sequential partitioning on all four variables further raises the explanatory power of earnings for returns from approximately 8.7% to 11.1%. We employ novel graphical methods that clearly demonstrate the appeal of our simple approach to examining convexity in the returns-earnings relation.

These results provide strong support for our four partitioning variables and the particular sequence in which we examine them. However, we emphasize that our approach is not the only insightful way to identify differences in optionality and thus convexity. To illustrate this point, we estimate convexity for two alternative partitions of observations suggested by the empirical accounting literature, again limiting the analysis to high volatility observations. First, we show that convexity is higher during the technology boom and hot equity market of mid-late 1990s than during the remainder of our 1974-2009 sample period. Second, we show that convexity generally is higher for earlier lifecycle stage firms using Dickinson's (2011) cash-flow-statement-classification-based proxy for lifecycle stage modified to treat R&D expenditures as investing rather than operating cash outflows.

The values of both left- and right-tail options increase with volatility. In this sense, convexity reflects the positive implications of volatility from the perspective of equityholders. To demonstrate this idea, we predict firms' future equity and debt financings based on our convexity measures, controlling for a number of predictor variables including return volatility,

[5] Freeman and Tse's (1992) regressions differ in various significant ways from ours. They use *quarterly* data for *abnormal* returns (measured from two days after the prior earnings announcement to one day after the current earnings announcement) and deflated *unexpected* earnings (measured relative to the median analyst earnings forecast in the last month of the quarter). We use annual data for raw returns and deflated earnings.

the number of years of consecutive losses, and financing deficit (Frank and Goyal 2003).

We find that all four convexity measures are positively (negatively) associated with firms' future equity (debt) financings, ceteris paribus. These results extend finance research showing that more volatile firms generally are likelier to issue equity than debt (Myers 1977, Leary and Roberts 2005, and Cotei and Farhat 2011). We also predict and find that left-tail convexity strengthens both the positive effects of loss sequence for equity issuances and the negative effects of loss sequence for debt issuances. These results suggest that our convexity measures have general usefulness for research in accounting and finance, which generally focuses on the negative implications of volatility.[6]

For example, recent research on bankruptcy prediction highlights the importance of supplementing accounting-based measures of distress with market-based measures of volatility (Vassalou and Xing 2004, Hillegeist et al. 2004, Franzen et al. 2007, and Campbell et al. 2008). Volatility accentuates firms' downside risk by increasing the probability that their assets are worth less than the face value of their debt and other fixed claims. Our findings indicate that our convexity measures capture volatility's positive effect on the value of continuation options. This accentuation of upside risk lessens the likelihood of firm failure even in the presence of continued losses by increasing financing opportunities. Our preliminary results in a separate project indicate that convexity is significantly negatively associated with the probability of bankruptcy controlling for other predictors of bankruptcy.

The remainder of the paper is organized as follows. Section 2 summarizes the related literature, describes continuation options, and develops our hypotheses about the determinants and implications of returns-earnings convexity. Section 3 describes our sample and variables

[6] A few exceptions exist to this statement, typically papers examining settings where volatility is a pure positive. For example, the literature on hybrid financing instruments such as convertible debt (Mayers 1998) recognizes the value of volatility to purchased options embedded in these instruments.

and reports descriptive statistics. Section 4 describes our spline regression and partitioning approach to estimating convexity and reports the results of these estimations. Section 5 uses our measures of convexity to predict future equity and debt issuances. Section 6 concludes.

[2] Related literature, continuation options, and hypotheses

[a] Literature on returns-earnings convexity

Beaver, Clarke and Wright (1979) and Freeman and Tse (1992) document convexity in the returns-earnings relation, i.e., returns are less responsive to losses than to small-to-moderate gains; in fact, returns are close to insensitive to losses. Beginning with Hayn (1995), accounting researchers have proposed and provided support for various explanations for this convexity and how it differs across firms. While distinct, these explanations exhibit the unifying theme that the responsiveness of returns to losses is attenuated relative to the "normal" responsiveness of returns to gains.

Hayn (1995) explains convexity in terms of firms' real option to abandon underperforming assets or the firm as a whole each period.[7] Hence, the value of a properly managed asset or firm cannot fall below its abandonment (hereafter "liquidation") value, regardless of how unfavorable earnings are during the period.[8] Fischer and Verrecchia (1997) explain convexity in terms of the limited liability of equity. Because equity investors reap most or all of the benefits of good news but have limited downside risk, they respond more strongly to good than bad news.[9] Sufficiently bad news causes equityholders to put the firm's assets to

[7] Berger et al. (1996) provide extensive discussion and analysis of the abandonment option. Burgstahler and Dichev (1997) discuss firms' somewhat more general option to adapt the use of assets in place.
[8] Optimal exercise of the liquidation option occurs when the option is in the money by more than the value of the option to delay liquidation on the chance of subsequent good news (Dixit and Pyndyck 1994, Chapter 7).
[9] Core and Schrand (1999) show theoretically and provide empirical evidence that debt covenants influence the sensitivity of equity returns to earnings around covenant thresholds, because covenants effectively protect debtholders against the limited liability of equity.

debtholders.[10]

More recent papers focus on the heterogeneity of losses and concomitant differences in the responsiveness of returns to losses across observations. Joos and Plesko (2005) find that returns have no association with persistent losses early in their sample period, consistent with the firms involved being likely to exercise the abandonment option, and are negatively associated with these losses late in their sample period. They provide evidence that unrecognized economic assets resulting from the immediate expensing of the costs of R&D explain the negative association late in their sample period. Intuitively, bigger or more persistent losses attributable to investments in R&D indicate larger unrecognized economic assets. In contrast, Joos and Plesko find that returns are positively associated with transitory losses, consistent with the firms involved being unlikely to exercise the abandonment option.

Darrough and Ye (2007) provide evidence that many firms experience a series of losses but are unlikely to liquidate, because they have unrecognized R&D and sales-growth-related economic assets as well as equity and debt financing opportunities. Darrough and Ye also point out that firms with histories of losses that survive likely do so because their positive future prospects more than compensate for those losses. They find a negative association between market value and losses, controlling for book value, that attenuates as proxies for unrecognized economic assets and financing during the current and prior year are added to the model.

Figure 1, Panel A, visually summarizes the findings of prior research on the determinants of loss firms' left-tail convexity. For simplicity, this figure plots the market value of equity

[10] Fischer and Verrecchia's (1997) analysis is consistent with classical option pricing-based models of risky debt (e.g., Merton 1974). In these models, equity is characterized as a purchased call option (equivalently, assets in place plus a purchased put option) on assets in place with exercise price equal to the face value of debt and constant capitalization. The value of this call option is a convex function of the value of the underlying assets. Convexity is designated by the parameter "gamma," which is the change in the slope of this function (i.e., the change in the parameter "delta") as the value of the underlying assets changes. Readers may find it useful to think of convexity arising from right-tail continuation options as captured by a generalized gamma parameter that allows for the value of assets not yet in place to depend on the magnitude of gains.

against the "value of assets," which should be interpreted as the economic value of assets for Fischer and Verrecchia (1997) and as the book value of assets for the other papers. The convexity depicted in this figure should carry over to the returns-earnings relation, because returns (deflated EPS) equal dividends per share plus the first difference of price (book value of equity per share) divided by beginning price.

[b] Continuation options and returns-earnings convexity

Continuation options essentially are the upside analogue of Hayn's (1995) abandonment option. Each period, firms have the option to continue operations, to make new investments, and to raise financing. Subsequent periods typically involve further decisions of these types. When this is the case, continuation is a compound option.

Dixit and Pyndyck (1994, Chapters 10 and 11) model several examples of continuation options related to investments made over time. These examples illustrate that current investments or observable indicators of future profitability can be associated with unrecognized economic assets attributable to firms' continuation options rather than their assets in place. For example, Dixit and Pyndyck model firms' learning curves as their cumulative past production reducing their future costs and thus being "like an investment." Dixit and Pyndyck also model firms' early-stage investment as revealing information about the profitability of their subsequent investments and thus having "a shadow value beyond its direct contribution to the completion of the project."

Continuation options can yield convexity in either the left or right tail of the earnings distribution, albeit for different reasons and at different stages in firms' lifecycles. Darrough and Ye (2007) hypothesize and provide evidence that left-tail convexity results from firms with larger current investments in R&D having larger current period losses, due to the immediate expensing of the costs of R&D under FAS 2, *Accounting for Research and Development Costs* (Accounting Standards Codification 730-10-15), and larger future gains as these unrecognized economic assets pay off in future periods. This effect is more likely to apply to earlier-lifecycle-stage firms, whose earnings should be more depressed by the immediate expensing of R&D expenditures. The line representing Joos and Plesko (2005) and Darrough and Ye (2007) in Figure 1, Panel A, summarizes the effect of greater left-tail continuation options on returns-earnings convexity.

Right-tail convexity should result when current period gains indicate increases in the probability or extent of continuation and the benefits to continuation when it occurs. This should occur for the range of gains above the level for which liquidation is likely and below the level for which (maximal) continuation is inevitable. The literature on stage financing of real options suggests that these effects occur during the lifecycle stage in which uncertainty is resolved about the benefits of the firm's investments in real options. For example, Bergemann, Hege, and Peng (2011) find that these investments increase as favorable information arrives while projects are being developed. This should occur fairly early in a firm's lifecycle, but somewhat after the stage that yields left-tail convexity described above.

Figure 1, Panel B, depicts our predictions about the effect of greater right-tail continuation options on convexity. In this figure, we do not depict convexity falling or reversing for sufficiently large gains, but it should beyond the level of gains for which continuation becomes sufficiently likely or the gains become sufficiently transitory.

[c] Hypotheses about the determinants of returns-earnings convexity

[i] Control variable determinants

We begin by proposing hypotheses about two variables that we expect to be related to firms' optionality in general and the left-tail optionality examined by prior research, and thus to overall convexity and left-tail convexity, respectively. Because these variables do not relate *directly* to real continuation options, we refer to them as control variables. Despite these semantics, our use of these variables to test these hypotheses is new.

First, for any option to have time value (i.e., value attributable to the ability to delay exercise of the option), the value of the underlying assets must be volatile. Hence, we hypothesize, in alternative form:

H1A: All four convexity measures increase with volatility.

Following Hillegeist, Keating, Cram and Lundstedt (2004), we measure volatility as the standard deviation of daily returns. Surprisingly, we are unaware of any prior research explaining convexity in terms of volatility, so H1A does not appear to have any direct precedent. Somewhat analogously, however, Patatoukas and Thomas (2011) relate volatility to the slope coefficients in Basu's (1997) piecewise reverse regression framework.

Second, Hayn's (1995) abandonment option and Fischer and Verrecchia's (1997) limited liability of common equity left-tail explanations for convexity relate to the probability that the firm's assets are liquidated or put by equityholders to debtholders. We refer to this probability as liquidation likelihood, and hypothesize:

H1B: Left-tail convexity increases with liquidation likelihood.

We make no prediction about the association of liquidation likelihood with right-tail convexity. We measure high versus low liquidation likelihood as below- versus above-median stock price.[11]

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[11] We could have measured liquidation likelihood based on an estimate of the closeness of the firm's value in use to its liquidation value. Hayn (1995) uses an estimate of the replacement cost of property, plant, and equipment to

[ii] Test variable determinants

We now turn to hypotheses about two test variables related to real continuation options. The first test variable is investments in real options and the second is equity financing of such investments. These variables are motivated by the large literature in financial economics (e.g., Myers 1977, Trigeorgis 1993, Hall and Lerner 2010, Bergemann, Hege, and Peng 2011) showing that investments in real options typically are financed in stages (i.e., conditional on adequate success to date) using equity.[12]

We measure our first test variable, investment real optionality, as an indicator variable for above versus below 5% R&D-to-sales, the threshold employed by Demers and Joos (2007). Like most of the prior literature, we focus on R&D due to its importance and observability across firms, although we acknowledge it would be desirable to develop a broader measure of investments in real options. We expect investment real optionality to increase both left- and right-tail convexity. Joos and Plesko (2005) and Darrough and Ye (2007) show that R&D increases left-tail convexity. We expect investment real optionality to increase right-tail convexity for the reasons discussed in detail in Section 2.b, i.e., gains indicate increases in the probability or extent of and benefits to continuation of firms' investments in real options. Summarizing this discussion, we hypothesize that:

H2A: All four convexity measures rise with investment real optionality.

Our second test variable is firms' net issuances of equity. Because firms may issue

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proxy for the firm's liquidation value. We chose not do this because of our focus on R&D and other continuation options. These economic assets often have significant liquidation value (e.g., for saleable patents and other legally protected intellectual property), but that value is very difficult to estimate independently of the firm's share price.

[12] Papers in this literature typically are couched in the context of venture capital, but the authors usually indicate the far wider applicability of stage financing of real options such as R&D. For example, Bergemann, Hege, Peng (2011, p. 2) state the "venture capital industry is a powerful example of the importance of these feedback effects in the financing of innovation. But similar issues also arise for innovative projects within large organizations or in publicly funded research."

equity for various reasons, we focus our analysis of this variable on firms that are likely to need financing for their investments in real options, i.e., firms with high investment real optionality.

Darrough and Ye (2007) show that loss firms' ability to raise debt and equity capital when financing deficits arise, one type of continuation option, increases left-tail convexity. However, the aforementioned financial economics literature shows that equity is the most natural form of external financing for firms investing in real options. This is particularly true for firms whose economic assets are primarily intangible and thus serve poorly as collateral (e.g., Carpenter and Petersen 2002 and Brown, Fazzari, and Petersen 2009), such as R&D-intensive firms. Such firms cannot handle debt well and, moreover, investors in these firms generally will want to participate in the upside. We refer to positive net issuances of equity in a year as market continuation (of investments in real options) and hypothesize:

H2B: All four convexity measures rise with market continuation.

We expect the strength of the implications of investment real optionality (the test variable in H2A) and market continuation (the test variable in H2B) for left- and right-tail convexity to vary depending on liquidation likelihood. Specifically, we expect higher (lower) liquidation likelihood to increase the strength of the effects of these options on left-tail (right-tail) convexity and hypothesize:

H2C: The effects of investment real optionality and market continuation on left-tail (right-tail) convexity are stronger for firms with higher (lower) liquidation likelihood.

[d] Hypotheses about the implications of convexity for future equity and debt financings

The values of both left- and right-tail options rise with volatility. In particular, left-tail convexity captures losses in firms for which liquidation is likely or losses that are otherwise nonproblematic. Right-tail convexity captures the probability or extent of and benefits to continuation. In this sense, convexity reflects the positive implications of volatility from the perspective of equityholders.

To demonstrate this idea, we propose hypotheses about how our convexity measures predict firms' future equity and debt financings. In testing these hypotheses, we control for stock return volatility, financing deficit, leverage, firm size, and number of years of consecutive losses. The inclusion of volatility is important because it captures the negative implications of volatility thereby allowing our convexity measures to capture the positive implications of volatility. Loss sequence also is an important control because it is inherently related to left-tail options.

Consistent with our earlier discussion of the financial economics literature, we expect equity to be the most natural type of financing for firms' investments in real options, more so for firms that are volatile and primarily hold intangible assets that serve poorly as collateral. For this reason, we hypothesize that all four of our convexity measures are significantly positively (negatively) associated with firms' future equity (debt) financings.

H3A: All four convexity measures are positively associated with future equity issuances and negatively associated with future debt issuances.

We expect firms with longer loss sequences to be less likely to issue debt and, if their continuation options are sufficiently strong, more likely to issue equity. Like liquidation likelihood, loss sequence pertains primarily to left-tail options. Hence, we hypothesize that firms with stronger left-tail convexity will be more likely to issue equity and less likely to issue debt in the presence of a sequence of losses.

H3B: Left-tail convexity strengthens the negative association between loss sequence and debt issuances and strengthens the positive (or weakens the negative) association between loss sequence and equity issuances.

[3] Data and descriptive statistics

We obtain annual financial reporting and year-end price data from Compustat and daily returns data from CRSP. We define all variables, providing the Compustat variable names involved, in Appendix A. We chose our data availability requirements to correspond to those in Hayn (1995) and other relevant prior research. We require each observation to have the data necessary to calculate: (1) earnings before extraordinary items per share for the year (X_t) divided by price at the beginning of the year (P_{t-1}), (2) returns for the 12-month period commencing with the beginning of the fourth month of the year (R_t), and (3) the standard deviation of daily returns over the same 12-month period (*stdret*). We also require each firm-fiscal year observation to have a share price above $2 both at the beginning and end of the year, in part because we deflate earnings by price and in part because low-price firms raise various significant economic issues (e.g., illiquidity) that could affect the behavior of returns. This requirement naturally reduces the percentage of loss firms in the sample, however. The resulting full sample contains 142,071 firm-year observations, representing 15,575 unique firms over the period 1974-2009.[13]

Our analyses include the following additional variables either to partition the sample to estimate convexity or to predict future financings. We do not require these variables to be available for observations to be included in the full sample, but they generally are available. The additional partitioning variables are: the indicator variable *high_rd*, which takes a value of one when R&D divided by sales exceeds 5% and zero otherwise (Demers and Joos 2007);[14] and the indicator variable *equity_issue*, which takes a value of one when net equity issuances in the year

[13] The entire sample period is after the incorporation of NASDAQ firms into Compustat.
[14] We assume R&D is zero if it is missing on Compustat. If a firm has R&D that is missing on Compustat, it almost surely is immaterial, because FAS 2 requires separate disclosure of material R&D.

are positive and zero otherwise (Frank and Goyal 2003).[15] The additional financing-prediction variables are the indicator variable *debt_issue*, which takes a value of one when net debt issuances in the year are positive and zero otherwise (Frank and Goyal 2003); the number of consecutive years of losses up to a maximum of six (*loss_seq*); long-term debt divided by the market value of equity (*leverage*); financing deficit (*fdeficit*) (Frank and Goyal 2003), see Appendix A for the involved definition; and the natural logarithm of the market value of equity (*log_mve*). We winsorize all continuous variables at the 1% and 99% levels.

Table 1 reports descriptive statistics for these variables for two partitions of the full sample. Panel A partitions the full sample into profit ($X_t/P_{t-1} > 0$) and loss ($X_t/P_{t-1} \leq 0$) observations. Panel B partitions the full sample into the top and bottom terciles of *stdret* each year. The panels report variable means, medians, and standard deviations for the observations in each partition, differences in these descriptive statistics across the partitions, and corresponding significance levels.

Not surprisingly, the descriptive statistics for profit and loss observations reported in Panel A differ significantly across all variables. Based on differences in means, compared to profit observations loss observations have poorer historical performance (lower R_t) and higher liquidation likelihood (lower P_{t-1}), are smaller (lower *log_mve*), more volatile (higher *stdret*), and more in need of financing (higher *fdeficit*), are currently more levered (higher *leverage*) but

[15] As discussed in Appendix A, Frank and Goyal's (2003) measure of net equity issuance uses cash flow statement data for both the inflow from issuances of equity and the outflow for repurchases of equity. We use their measure in part because it is used in most of the recent financing prediction literature (e.g., Brown, Fazzari and Petersen 2009) and in part because we do not want to classify firms that reissue repurchased stock in satisfaction of exercises of employee stock option grants as exhibiting market continuation. We note, however, that Darrough and Ye (2007) use only the inflow from issuances of equity and Fama and French (2005) propose two measures based on the change in book value and number of shares outstanding. While these four approaches yield highly correlated measures of equity issuance, Frank and Goyal's cash-flow-statement based measure provides the lowest estimate of equity issuance each year, with 49% of our sample firm-years having positive equity issuance. In contrast, using Fama and French's book-value (number of shares) based approach, 71% (65%) of our sample firm-years have positive equity issuance.

more likely to issue equity (higher *equity_issue* and lower *debt_issue*), and have higher investment real optionality (higher *high_rd*). Loss observations' mean *loss_seq* is 2.3 years.

Similarly significant differences in the descriptive statistics exist for high and low volatility observations reported in Panel B. Based on differences in means, compared to observations in the bottom tercile of *stdret*, observations in the top group have significantly poorer historical performance (lower X_t/P_{t-1} and higher *loss_seq*) and higher liquidation likelihood (lower P_{t-1}), are smaller (lower *log_mve*) and more in need of financing (higher *fdeficit*), are more likely to issue equity instead of debt (higher *equity_issue* and lower *debt_issue*), and have higher investment real optionality (higher *high_rd*). Partitioning on *stdret* yields skewness and/or heterogeneity in some variables; observations in the top tercile of *stdret* have significantly higher mean and lower median R_t and significantly higher mean but lower median *leverage*.

Collectively, the two panels of Table 1 indicate that investment real optionality and market continuation are more strongly present in loss and high volatility firms.

Table 2 presents univariate Pearson (below the diagonal) and Spearman (above the diagonal) correlations for the same variables as in Table 1 for the full sample. The correlations of the four partitioning variables with each other, with *loss_seq*, and with *equity_issue* and *debt_issue* all have the signs one would expect. For each of the first three partitioning variables, the sign of the variable's correlation with *equity_issue* is opposite to the sign of its correlation with *debt_issue*. For example, the correlation of *high_rd* with *equity_issue* is 0.20 and with *debt_issue* is −0.10. Generalizing, the correlations indicate that firms with greater optionality—investment real optionality in particular—issue equity rather than debt.

In order to provide insight into how well the volatility and liquidation likelihood partitions capture the left-tailed options examined by prior research, we ranked observations each year into deciles based on *stdret* and P_{t-1} and calculated the frequency of losses in the following year for each possible intersection of decile rankings. Figure 2 graphically presents these loss frequencies, which indicate that volatility and liquidation likelihood have considerable explanatory power for loss frequency. For example, the intersection of the lowest volatility and highest price deciles in our sample has a loss frequency of approximately 3%, whereas the intersection of the highest volatility and lowest price deciles in our sample has a loss frequency of approximately 47%.[16] Moreover, loss frequency rises close to monotonically with volatility decile, holding price decile constant, and falls close to monotonically with price decile, holding volatility decile constant.[17] This suggests that our two control variables should capture a considerable portion of the left-tail options and thus convexity examined in prior research.

[4] Convexity measures and estimation

[a] Spline regression, measures of convexity, and graphical methods

To calculate our four convexity measures, we regress R_t on X_t/P_{t-1} using a spline regression with a single knot located at $X_t/P_{t-1} = x^*$. This spline regression model is:

$$R_t = \alpha_{\mathrm{SPL}} + \beta_{\mathrm{L}}\left(\frac{X_t}{P_{t-1}}\right) + \beta_{\mathrm{IR}} \max\left(\frac{X_t}{P_{t-1}} - x^*,\ 0\right) + \varepsilon_t. \qquad \text{[1-SPL]}$$

β_{IR} is the incremental slope coefficient for values of X_t/P_{t-1} above x^*; we refer below to the total coefficient β_{R}, which equals $\beta_{\mathrm{L}} + \beta_{\mathrm{IR}}$. Rather than impose the horizontal location of the knot at $x^* = 0$, which would distinguish profits from losses as in Hayn (1995), we estimate equation

[16] The loss frequencies in Figure 2 would rise significantly, particularly for the groups with high loss frequency, if we did not require firms to have share price above $2.

[17] Durtschi and Easton (2005) and Patatoukas and Thomas (2011) also demonstrate the association of loss frequency with stock return volatility and price.

[1-SPL] iteratively using maximum likelihood estimation to identify the value of x^* that minimizes the total sum of squared residuals. The estimate of x^* generally is considerably less than zero in our samples. It is important to allow the knot to move both horizontally and particularly vertically for each of our samples to obtain accurate measures of convexity; otherwise convexity might vary across the samples solely due to differences in the locations of the true knots.[18]

In order to calculate one of our convexity measures, we also estimate the analogous OLS regression equation:

$$R_t = \alpha_{\mathrm{OLS}} + \beta_{\mathrm{OLS}}\left(\frac{X_t}{P_{t-1}}\right) + \varepsilon_t . \qquad \text{[1-OLS]}$$

We calculate four measures of convexity based on these regressions. The first two measures capture overall convexity. *rl_slope_diff* equals the estimate $\hat{\beta}_{\mathrm{R}} - \hat{\beta}_{\mathrm{L}}$ from the spline regression. *ols_spl_dist* equals the vertical distance between the fitted value of the OLS regression at the spline knot x^* (i.e., $\hat{\alpha}_{\mathrm{OLS}} + \hat{\beta}_{\mathrm{OLS}} x^*$) and the vertical location of the estimated spline knot, (i.e., $\hat{\alpha}_{\mathrm{SPL}} + \hat{\beta}_{\mathrm{L}} x^*$), which equals $\left(\hat{\alpha}_{\mathrm{OLS}} - \hat{\alpha}_{\mathrm{SPL}}\right) + \left(\hat{\beta}_{\mathrm{OLS}} - \hat{\beta}_{\mathrm{L}}\right) x^*$. Absent convexity, *rl_slope_diff* and *ols_spl_dist* would both equal zero. *ols_spl_dist* differs from *rl_slope_diff* because $\hat{\alpha}_{\mathrm{OLS}} - \hat{\alpha}_{\mathrm{SPL}}$ increases with the dispersion of X_t/P_{t-1}. For this reason, *ols_spl_dist* indicates the benefit of estimating spline rather than OLS regressions more completely than does *rl_slope_diff*.

The remaining two convexity measures are one-tailed and derived from the spline

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[18] As discussed in Sections 4.c.iv and 6, the vertical location of the knot moves considerably with lifecycle stage. This unexpected finding is worthy of future research. Such research will need to consider accounting biases and other factors that are tangential to this study.

regression. *neg_l_slope* equals $-\hat{\beta}_{\mathrm{L}}$, because a lower coefficient to the left of the knot indicates greater convexity. *r_slope* equals $\hat{\beta}_{\mathrm{R}}$.

The spline regression output provides standard errors—calculated clustering by both firms and time—for three of the four convexity measures: *rl_slope_diff*, *neg_l_slope*, and *r_slope*. We use bootstrapping to calculate the standard error for *ols_spl_dist*.[19]

We employ novel graphical methods to demonstrate the goodness of fit of our simple approach to estimating convexity in the returns-earnings relation. For each of the two samples in each partition, we sort observations into bins based on the level of X_t/P_{t-1}. These bins generally have width of 0.01 for values of X_t/P_{t-1} between −0.10 and 0.20 and width of 0.04 for the considerably less frequent observations of X_t/P_{t-1} below or above this central range.[20] Bubbles depict both the mean of R_t (the vertical center of the bubbles) and the percentage of the total observations in both samples (the size of the bubbles) in each X_t/P_{t-1} bin for each sample in the partition. We overlay the estimated spline regressions for each sample in the partition on the bubble plots to demonstrate how closely the spline regressions follow the path of the bubbles for the partition.

[b] Full sample

We first demonstrate our spline and OLS estimations, convexity measures, and graphical methods for the full sample (n = 142,071). Table 3 reports the estimations and associated convexity measure calculations. Figure 3 depicts the graphical methods and, for convenience,

[19] We randomly draw 500 samples with replacement. The size of each random sample is 10% of the underlying sample for the full sample and for the first two partitions based on volatility and liquidation likelihood. The sample size is 100% of the sample for the third and fourth partitions based on investment real optionality and market continuation, respectively, because these underlying samples are smaller than the others due to the sequential partitioning. We compute the standard error of *ols_spl_dist* across the resulting 500 values.

[20] Because of the fewer observations in the market continuation (i.e., fourth sequential) partition, we double the bin widths for this partition.

also reproduces the calculated convexity measures.

As reported in Table 3, for the spline regression, the estimate of $neg\_l\_slope = -\hat{\beta}_{\mathrm{L}}$ is 0.063 (t = 2.4) and the estimate of $r\_slope = \hat{\beta}_{\mathrm{R}}$ is 2.011 (t = 85.7). The difference of these coefficients, $rl\_slope\_diff = \hat{\beta}_{\mathrm{R}} - \hat{\beta}_{\mathrm{L}}$ is 2.074 and significant (t = 57.2). The estimated spline knot is located at $x^* = -0.016$. Given OLS estimates of $\hat{\alpha}_{\mathrm{OLS}} = 0.133$ and $\hat{\beta}_{\mathrm{OLS}} = 0.883$, *ols_spl_dist* equals:

$$\left(\hat{\alpha}_{\mathrm{OLS}} - \hat{\alpha}_{\mathrm{SPL}}\right) + \left(\hat{\beta}_{\mathrm{OLS}} - \hat{\beta}_{\mathrm{L}}\right) x^* = \left(0.133 + 0.006\right) + \left(0.883 + 0.063\right)\left(-0.016\right) = 0.125$$

(t = 802.5). Hence, the full sample exhibits convexity based on both overall measures. Absent a benchmark, we cannot say much about left- or right-tail convexity in this single sample, but we will when we compare these one-tail convexity measures across the groups in each partition in the following section.

Figure 3 contains the bubble plot with overlaid estimated spline regression line and (in this plot only, to demonstrate the calculation of *ols_spl_dist*) the estimated OLS regression line. Despite having only one knot, the estimated spline regression hugs the bubbles tightly. In comparison, the estimated OLS regression fits the returns-earnings relation poorly, yielding fitted values of R_t that are too high for central values of X_t/P_{t-1} and too low for extreme values of X_t/P_{t-1}. The vertical line representing the distance between the OLS fitted value at x^* and the spline knot y^* equals *ols_spl_dist*.

To illustrate the goodness of fit of our simple one-knot spline, Figure 3 overlays two more complex models that capture the concavity of the returns-earnings relation for sufficiently large earnings documented by Freeman and Tse (1992). The first is a two-knot spline with both knots identified by the iterative maximum likelihood methods described above. The second is a nonlinear logistic function that Freeman and Tse (1992) report yields similar results as the arctan function used in their tabulated results. The left segments of the one-knot and two-knot splines are nearly identical. The right segment of the one-knot spline misses some minor concavity on the far right of the range of earnings that is captured by the middle and right segments of the two-knot spline. The improvement in explanatory power from this additional complexity is minimal, however; the one-knot (two-knot) spline yields an R^2 of 8.7% (8.8%). The left segment of the one-knot spline fits the data better than the logistic function, which cannot generate a negative slope. The right segment of the one-knot spline follows the logistic function closely for all but the right-most two earnings bins, for which some firms clearly have transitory earnings that pull the logistic function down. Because the one-knot spline fits the left segment better, it has higher explanatory power than the logistic function, with an R^2 of 8.7% versus 8.4%. These results provides additional support for the one-knot spline as a simple model that provides a good fit to the data and thus accurate measures of convexity over the range of earnings for which continuation options are likely to be of primary importance.

Summarizing, consistent with prior research, Table 3 and Figure 3 indicate that investors respond more to gains than to losses of the same absolute magnitude—in fact, the response to losses is slightly negative—yielding a convex relation between returns and earnings. Below, we explore the determinants of this convexity. In particular, does convexity stem from firms' right-tail continuation options as well as the left-tail options examined in prior research?

[c] Partitioning on the determinants of returns-earnings convexity

[i] Partitioning approach (the "optionality tree")

Table 4, Panel A depicts our sequential partitioning of the full sample each year to estimate convexity. For reasons evident from inspection of the panel, we refer to this depiction as the “optionality tree.” The panel reports the number of observations and the frequency of losses for each node (hereafter, “cell”) in the tree.

As depicted in the panel, we first partition observations each year into terciles based on volatility (*stdret*). We then partition observations in each volatility group each year into low and high liquidation likelihood groups (above- and below-median P_{t-1}, respectively). We then partition observations in each volatility/liquidation likelihood group each year into high and low investment real optionality (*high_rd* = 1 and = 0, respectively). Finally, we partition each volatility/liquidation likelihood/investment real optionality group each year into net positive and net negative equity issuances (*equity_issue* = 1 and = 0, respectively). The three volatility, two liquidation likelihood, two investment real optionality, and two market continuation groups in the partitions yield 24 possible paths through the optionality tree, which we number in Panel A.

The volatility and liquidation likelihood partitions yield equal-size groups, while the investment real optionality and market continuation partitions do not. The number of observations in the cells of the optionality tree naturally declines with each sequential partition. The loss frequencies of the subsamples vary across the cells consistent with the descriptive analyses in Tables 1 and 2 discussed in Section 3. Higher volatility, higher liquidation likelihood, and higher investment real optionality are all consistently associated with higher loss frequency. With a slight exception for path 23 versus path 24, the “Yes” market continuation groups display a higher loss frequency than the corresponding “No” groups.

[ii] Non-sequential partitions

Because of the sequential nature of our analysis, earlier partitions in the optionality tree will have both larger group sizes and precedence in capturing optionality and thus likely show the strongest effects on convexity. We believe that our particular sequence proceeds naturally from the most general to the most specific types of options and clearly demonstrates the incremental effects of right-tailed continuation options beyond left-tail options. However, to give a sense for how the results might change were we to alter the partitioning sequence, Table 4, Panel B reports the four convexity measures for the groups formed in separate (i.e., nonsequential) partitionings based on each of the four partitioning variables. The panel reports the magnitude and significance of the differences in the convexity measures across the groups in each partitioning. The panel indicates that each partitioning variable has significant effects on each convexity measure, consistent with hypotheses H1A, H1B, H2A, and H2B.

We note two interesting aspects of the results reported in this panel. First, the partitioning variables differ predictably in the relative strengths of their effects on left-tail versus right-tail convexity. Liquidation likelihood has larger effects on left-tail than on right-tail convexity. The other three partitioning variables have larger effects on right-tail than on left-tail convexity, with investment real optionality having the largest effect on right-tail convexity, followed by market continuation and then volatility. Second, the relative magnitudes of the overall convexity measures vary across the partitioning variables. In particular, *rl_slope_diff* is relatively larger than *ols_spl_dist* for the investment real optionality and market continuation partitioning variables, consistent with stage financing reducing the dispersion of earnings and thus *ols_spl_dist* for the high optionality groups in these partitions.

[iii] Sequential partitions

In this section, we focus on the specific paths through the optionality tree for which we expect optionality to be most evident or interesting. The regressions and convexity measures for the paths we examine are summarized in Panels A–D of Table 5 and depicted in the corresponding panels of Figure 4.[21] For completeness, we tabulate the convexity measures for all 24 paths in Table 6.

<u>Volatility</u>

The first partitioning variable we examine is volatility, for which we compare the high and low terciles. We report and depict the results in Panels A of Table 5 and Figure 4, respectively. Hypothesis H1A predicts that higher volatility observations have higher left- and right-tail convexity. Consistent with these predictions, Panel A of Table 5 reports that all four convexity measures are significantly higher for the high volatility group than the low volatility group. Panel A of Figure 4 illustrates this higher convexity, as well as upward shift, in the returns-earnings relation for the high volatility group.

This figure depicts two additional interesting effects. First, the combined sample spline regression is dominated by the high volatility group, particularly for the segment to the left of the spline knot. This is due to the higher earnings dispersion in the high volatility group, as indicated by the size of the bubbles for the extreme X_t/P_{t-1} bins. This higher earnings dispersion explains the much larger value of *ols_spl_dist* for this group. Second, inspection of the bubbles indicates that right-tail convexity falls off in the high volatility group for values of X_t/P_{t-1} above approximately 0.2. Presumably this reflects either a high probability of continuation or transitory gains.

Because volatility is central to all optionality, all of the comparisons we discuss

[21] Note that the panels of Figure 4 are slightly different from Figure 3. To avoid clutter we omit the OLS and nonlinear exponential fitted lines, although we do depict *ols_spl_dist* with vertical lines from the spline knot to the fitted OLS line. In addition, we depict the spline regression for the combination of the two groups with the dotted line to illustrate how the combined results typically are dominated by the group with higher earnings dispersion.

subsequently involve further partitioning of the high volatility group.

Liquidation likelihood

The next partitioning variable we examine is liquidation likelihood, for which we compare high volatility and high liquidation likelihood (below-median price) to high volatility and low liquidation likelihood (above-median price) observations. We report and depict the results in Panels B of Table 5 and Figure 4, respectively. Hypothesis H2B predicts that high liquidation likelihood observations have higher left-tail convexity than low liquidation likelihood observations. Consistent with these predictions, Panel B of Table 5 reports that the left-tailed convexity measure is significantly higher at the 10% level for the high liquidation likelihood group. While we did not predict this, the right-tailed convexity measure is significantly lower for the high liquidation likelihood group.[22] This unexpected finding may be consistent with Dhaliwal and Reynolds' (1994) finding that earnings response coefficients decrease with default risk as measured by bond ratings. As a consequence, the difference in the overall *rl_slope_diff* convexity measure across the groups is insignificant. The overall *ols_spl_dist* convexity measure is higher for the high liquidation likelihood group; this results from this group's greater earnings dispersion, particularly in the left tail of the distribution.

Panel B of Figure 4 depicts the higher left-tail convexity of the high liquidation likelihood group. The figure also indicates that the segment to the left of the spline knot for the combined sample is dominated by the high liquidation likelihood group, due to the higher dispersion of earnings in the left tail of its distribution for this group. Finally, the figure depicts the convergence of the segments to the far right of the spline knot for the two liquidation

[22] This results from all observations in this analysis being in the highest volatility group. As indicated in Table 4, Panel B, without control for volatility, liquidation likelihood is positively associated with right-tail convexity.

likelihood groups. This convergence may stem from very high earnings yielding a low likelihood of liquidation even for firms with high beginning liquidation likelihoods.

Collectively, the evidence for the sequential volatility and liquidation likelihood partitions suggests these partitions successfully identify increased overall optionality/convexity and increased left-tail optionality/convexity, respectively.

Investment real optionality

As predicted in hypothesis H2A, investments in real options (R&D) give rise to both left- and right-tail options. To test this hypothesis, we compare high investment real optionality (*high_rd* = 1) and low investment real optionality (*high_rd* = 0) observations. Because hypothesis H2C predicts that the left-tail options should be more (less) important than the right-tail options for high (low) liquidation likelihood observations, we conduct these comparisons for two distinct underlying samples, one containing high volatility and low liquidation likelihood observations and the other containing high volatility and high liquidation likelihood observations. We report and depict the results for the former (latter) underlying sample in Panels C1 (C2) of Table 5 and Figure 4, respectively.

For the underlying sample of high volatility and low liquidation likelihood observations, Panel C1 of Table 5 indicates that high investment real optionality observations have significantly higher overall convexity and right-tail convexity measures, but an insignificantly different left-tail convexity measure, than the low investment real optionality observations.

For the underlying sample of high volatility and high liquidation likelihood observations, Panel C2 of Table 5 indicates that high investment real optionality observations have significantly higher overall *rl_slope_diff*, left-tail (at the 10% level), and right-tail measures than low investment real optionality observations. In contrast, the overall *ols_spl_dist* is significantly lower for the high investment real optionality observations. Inspection of Panel C2 of Figure 4 reveals that the reason for this is the considerably lower dispersion of earnings for the high investment real optionality observations in the high liquidation likelihood underlying sample. This is consistent with stage financing of investments in real options being particularly important for firms with high liquidation likelihood.

Collectively, the results in Panels C1 and C2 of Table 5 and Figure 4 are consistent with hypothesis H2A that investment real optionality yields both left- and right-tail convexity.[23] These results are also consistent with hypothesis H2C that left-tail (right-tail) options are more important for high (low) liquidation likelihood observations. The relative magnitudes of the difference of *neg_l_slope* and *r_slope* across the two panels are particularly revealing in this regard. For example, the 0.064 difference in *neg_l_slope* for the high versus low real investment real optionality groups with low liquidation likelihood and high volatility in Panel C1 is about a third of the 0.180 difference in this left-tail convexity measure for the for the high versus low real investment real optionality groups with high liquidation likelihood and high volatility in Panel C2. Conversely, the 1.928 difference in *r_slope* for the high versus low real investment real optionality groups with low liquidation likelihood in Panel C1 is more than double the 0.812 difference in this right-tail convexity measure for the high versus low real investment real optionality groups with high liquidation likelihood in Panel C2.

<u>Market continuation</u>

As predicted in hypothesis H2B, market continuation (equity financing) of investments in real options give rise to both left- and right-tail options. To test this hypothesis, we compare

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[23] The left-tail optionality results are consistent with those in Joos and Plesko (2005) and Darrough and Ye (2007).

market continuation (*equity_issue* = 1) to no market continuation (*equity_issue* = 0) observations for the underlying sample of observations with high volatility, high liquidation likelihood, and high investment real optionality. We report and depict the results in Panels D of Table 5 and Figure 4, respectively. While hypothesis H2C predicts that the left-tail options should be more (less) important than the right-tail options for high (low) liquidation likelihood observations, to conserve space we do not conduct parallel analyses as we did for the investment real optionality partitions. Similar insights would result if we did so. Because the observations we examine have high liquidation likelihood, left-tail convexity should be increased and right-tail convexity diminished in the underlying sample we do examine. As this is the fourth sequential partition, the reduced sample size (1,346 observations in the no market continuation group) diminishes statistical significance.

Consistent with hypothesis H2B, Panel D of Table 5 reports significantly larger *rl_slope_diff* (10% level), *ols_spl_dist*, and *neg_l_slope* for the market continuation observations than for the no market continuation observations. The difference of *r_slope* is also positive and fairly large but insignificant, likely due to the reduced sample size. Nonetheless, Panel D of Figure 4 again offers visual evidence of increased convexity and an upward shift in the return-earnings relation associated with market continuation. The figure indicates that the combined sample is dominated by the market continuation observations; this is for the simple reason that these observations are approximately four times as numerous as the no market continuation observations in this underlying sample. This sample tends to need financing and equity is the natural type of financing.

<u>Summary</u>

For completeness, Panel A of Table 6 reports the four convexity measures for each of the cells along each of the 24 paths of the optionality tree. Inspection of this panel indicates that the convexity measures generally behave as expected across the partitions. Panel B of Table 6 illustrates the increased explanatory power gained by both incorporating average convexity (i.e., estimating spline regressions with a single knot) and sequentially partitioning based on cross-sectional indicators of convexity. Estimating a spline regression (as opposed to a basic OLS regression) model on the full sample raises the explanatory power (R^2) from 5.1% to 8.7%, consistent with the findings of Freeman and Tse (1992). Allowing convexity to vary across the 24 paths created by our four partitioning variables further increases explanatory power from 8.7% to 11.1%.[24]

[iv] Alternative partitioning variables

Despite this substantial increase in explanatory power, we emphasize that our partitioning variables and the sequence in which we examine them are not the only insightful way to identify differences in continuation options or to demonstrate the resulting variation in convexity across observations. To illustrate this point, Table 7 reports the estimated convexity measures and tests of differences in these measures for two alternative partitions of observations suggested in the empirical accounting literature: (1) the 1995-1999 technology boom/hot equity markets versus the remainder of our sample (Billings and Jennings 2011) and (2) lifecycle stage (Dickinson 2011). As with the second through fourth partitions in Table 6, we limit these analyses to high volatility observations due to the critical importance of this variable.

First, continuation options should be stronger during technology booms, when investments in real options are more common. This is particularly true for booms that give rise

[24] We calculate each incremental R^2 as the additional regression sum of squares explained by the model expansion under consideration divided by the total sum of squares residuals for the OLS regression for the full sample.

to hot equity markets, when equity financing of these investments is more readily available. A striking example of a technology boom with hot equity markets occurred during 1995-1999. We predict that both left- and right-tail convexity are higher during that period than the remainder of our 1974-2009 period. Consistent with this prediction, Panel A of Table 7 reports that left-tail convexity (at the 10% level) and both overall convexity measures are significantly higher in the bubble period; right-tail convexity is also higher but not significantly so.

Second, continuation options should be stronger for firms in the relatively early lifecycle stages in which uncertainty is resolved about the benefits of investments in real options. We identify lifecycle stage using a modification of Dickinson's (2011) cash-flow-statement-classification-based proxy, which classifies observations into introduction, growth, mature, shake-out, and decline stages based on the signs of their operating, investing, and financing cash flows. To better capture investments in real options, our focus, we reclassify R&D expenditures as investing rather than operating cash outflows. Appendix B details Dickinson's proxy, our modification of that proxy, and the modification's effects on lifecycle-stage classifications.

We generally predict that both left- and right-tail convexity decrease as firms proceed from the introduction to decline stages, with the following caveats. Left-tail convexity should rise with liquidation likelihood, which we expect to be high for the decline stage. Right-tail convexity may not decrease monotonically from the introduction stage to the growth or even mature phases; uncertainty about the probability/extent of and benefits to investments in real continuation options could be most resolved in any of these stages, depending on contextual (e.g., technological and competitive) factors beyond the scope of this preliminary analysis.

The results reported in Panel B of Table 7 are largely consistent with the general prediction, particularly regarding right-tail convexity. Right-tail convexity rises slightly from introduction stage to the growth stage, and then falls fairly sharply to the shake-out stage. Left-tail convexity generally falls from the introduction to shake-out stages, but then rises noticeably in the decline phase, presumably because of increased liquidation likelihood.[25]

[5] Implications of convexity for future equity and debt financings

Our convexity measures capture the value attributable to left-tail and/or right-tail options, and thus the positive implications of volatility, from the perspective of equityholders. For this reason, these measures should contain distinct and incrementally useful information beyond the measures of volatility employed in the empirical accounting and finance literatures, such as return volatility. To demonstrate this incremental usefulness, we examine how our convexity measures predict *equity_issue* and *debt_issue* in the following year (t + 1), controlling for the current year (t) values of the following variables: stock return volatility ($stdret_t$), financing deficit ($fdeficit_t$), leverage ($leverage_t$), firm size (log_mve_t), and loss sequence ($loss\_seq_t$). These control variables are defined in Section 3 and Appendix A.

Equity investors primarily benefit from upside realizations of risk, whereas debt investors primarily are hurt by downside realizations of risk. For this reason, equity investors desire convexity but debt investors are averse to it. Accordingly, hypothesis H3A predicts that both left- and right-tail convexity are positively associated with future equity issuances and negatively associated with future debt issuances. We expect firms with longer loss sequences to be less likely to issue debt and, if their continuation options are sufficiently strong, more likely to issue equity. Loss sequence pertains primarily to left-tail options. Hence, hypothesis H3B predicts

[25] While we did not predict this, interestingly, the vertical location of the spline knot varies considerably with lifecycle stage, being low for introduction stage observations, high for growth and mature stage observations, and intermediate for shakeout and decline stage observations. We believe these effects are worthy of future research. For example, we speculate that the low spline knot for introduction stage firms reflects the fact that they are valued in much the same way as out-of-the-money options. Ni (2007) and other finance research finds that out-of-the-money purchased call options yield low or negative average returns.

that firms with stronger left-tail convexity will be more likely to issue equity and less likely to issue debt in the presence of a sequence of losses.

In order to test these hypotheses, we predict $equity\_issue_{t+1}$ and $debt\_issue_{t+1}$ by estimating various nested versions of the following logistic regression models on the full sample:

$$\begin{aligned}
\Pr\left(equity_issue_{t+1}=1\right) = \alpha &+ \beta_1\left(stdret_t\right) + \beta_2\left(loss_seq_t\right) + \beta_3\left(loss_seq_t \times stdret_t\right) \\
&+ \beta_4\left(leverage_t\right) + \beta_5\left(fdeficit_t\right) + \beta_6\left(log_mve_t\right) \\
&+ \beta_7\left(ols_spl_dist\right) + \beta_8\left(loss_seq_t \times ols_spl_dist\right) \\
&+ \beta_9\left(rl_slope_diff\right) + \beta_{10}\left(loss_seq_t \times rl_slope_diff\right) \\
&+ \beta_{11}\left(neg_l_slope\right) + \beta_{12}\left(loss_seq_t \times neg_l_slope\right) \\
&+ \beta_{13}\left(r_slope\right) + \beta_{14}\left(loss_seq_t \times r_slope\right) + \varepsilon
\end{aligned} \qquad [2]$$

$$\begin{aligned}
\Pr\left(debt_issue_{t+1}=1\right) = \alpha &+ \gamma_1\left(stdret_t\right) + \gamma_2\left(loss_seq_t\right) + \gamma_3\left(loss_seq_t \times stdret_t\right) \\
&+ \gamma_4\left(leverage_t\right) + \gamma_5\left(fdeficit_t\right) + \gamma_6\left(log_mve_t\right) \\
&+ \gamma_7\left(ols_spl_dist\right) + \gamma_8\left(loss_seq_t \times ols_spl_dist\right) \\
&+ \gamma_9\left(rl_slope_diff\right) + \gamma_{10}\left(loss_seq_t \times rl_slope_diff\right) \\
&+ \gamma_{11}\left(neg_l_slope\right) + \gamma_{12}\left(loss_seq_t \times neg_l_slope\right) \\
&+ \gamma_{13}\left(r_slope\right) + \gamma_{14}\left(loss_seq_t \times r_slope\right) + \varepsilon.
\end{aligned} \qquad [3]$$

To avoid multicollinearity by construction, we estimate four versions of these models that contain the following convexity variables (all control variables are included in all models): (1) none, (2) only the *ols_spl_dist* variables, (3) only the *rl_slope_diff* variables, and (4) only the *neg_l_slope* and *r_slope* variables. We obtain the values of our four convexity measures (*rl_slope_diff*, *ols_spl_dist*, *neg_l_slope*, and *r_slope*) from the values reported for each of the 24 paths in the optionality tree in Panel A of Table 6.

We expect the signs on some of the control variables—in particular, *stdret*, *loss_seq*, and their interaction—to depend on whether or not we include the convexity variables in the model. In the absense of the convexity variables, *stdret* and *loss_seq* could capture the positive implications of volatility that we ascribe to the convexity variables. In the presence of the convexity variables, *stdret* and *loss_seq* should have more negative implications.

We expect firms with higher *stdret* to be more likely to issue equity ($\beta_1 > 0$) and less likely to issue debt ($\gamma_1 < 0$). Similarly, we expect firms with higher *loss_seq* to find it more difficult to issue debt ($\gamma_2 < 0$) but to be able to issue equity if their continuation options are sufficiently strong ($\beta_2 > 0$). To the extent that our convexity measures capture the positive implications of volatility, however, β_1 and β_2 could become negative. We interact *stdret* with *loss_seq* primarily because we also interact our convexity measures with *loss_seq*. We expect firms with high *stdret* and *loss_seq* to find it particularly difficult to issue debt ($\gamma_3 < 0$). We do not have an expectation for β_4; the combination of high *stdret* and *loss_seq*, while definitely favoring equity financing, could make any form of financing difficult to obtain.

We do not have expectations for *leverage*, because higher leverage indicates both that the firm successfully issued debt in the past and that it may have less debt capacity. We expect higher *fdeficit* to increase the likelihood of both equity ($\beta_5 > 0$) and debt ($\gamma_5 > 0$) issuance, all else being equal. We do not have expectations for *log_mve*, which we include to control for size.

Panel A (B) of Table 8 presents the results of predicting *equity_issue* in equation [2] (*debt_issue* in equation [3]). In each panel, the model reported in the first column includes only the control variables, the model reported in the second column includes the control variables and the *ols_spl_dist* convexity variables, the model reported in the third column includes the control variables and the *rl_slope_diff* convexity variables, and the model reported in the fourth column includes the control variables and the *neg_l_slope* and *r_slope* convexity variables. We report the area under the receiver operating characteristic ("ROC") curve to indicate the model goodness of fit. This statistic represents the estimated probability that the model ranks a randomly chosen actual issuance higher than a randomly chosen non-issuance. Random guessing yields an area under the ROC curve equal to 0.50, while perfect prediction yields a statistic of 1.00. Hosmer and Lemeshow (2000) state that an area under the ROC curve of 0.70–0.80 (above 0.80) indicates an acceptable (excellent) model.

We consider first the results for *equity_issue* in Panel A. The area under the ROC curve rises from 0.711 in the model with only the control variables to 0.779 in the model distinguishing left- and right-tail convexity. The models are all in Hosmer and Lemeshow's (2000) acceptable range, with the model that most fully captures convexity being close to the top of the range. Overall, the addition of the convexity variables yields a substantial improvement in fit.

The coefficients on the control variables generally are as expected or, absent expectations, interpretable. The coefficient on *stdret* is significantly positive in the model without convexity variables in the first column, but becomes significantly negative in the models with convexity variables in the second through fourth columns. Intuitively, in the latter models the convexity variables capture good variability desired by equityholders, while *stdret* captures undesirable volatility. In the former model, *stdret* primarily captures good variability. Likely for the same reasons, the coefficient on *loss_seq* is significantly positive in the model without convexity variables in the first column, but becomes significantly negative in the models with convexity variables in the second through fourth columns.

The coefficient on *leverage* is significantly negative in the first and second columns and significantly positive in the third and fourth columns. The negative coefficients are consistent with levered firms having debt and tending to issue more of the same. (*leverage* loads positively in all of the *debt_issue* models, as discussed below.) The positive coefficients appear when the convexity variables with the most explanatory power for *equity_issue* are added to the model.

We conjecture that once convexity is adequately controlled for, higher *leverage* indicates that additional financing should be equity to return the firm to an optimal capital structure. As expected, the coefficient on *fdeficit* is consistently significantly positive; this variable is highly significant as in the financing literature (Frank and Goyal 2003). The coefficient on *log_mve* is consistently significantly positive.

Consistent with hypothesis H3A, the coefficients on each of the four convexity measures are significantly positive. In the fourth column, the coefficient on *r_slope* is nearly double that of the coefficient on *neg_l_slope*, consistent with equity investors being more affected by right-tail options. Consistent with hypothesis H3B, we find that the coefficient on the interaction of *loss_seq* with *neg_l_slope* is significantly positive, consistent with left-tail convexity increasing the association of *loss_seq* with *equity_issue*. While we made no hypotheses in this regard, the coefficient on the interaction of *loss_seq* with *r_slope* is significantly positive.

We turn now to the results for *debt_issue* in Panel B. The area under the ROC curve rises from 0.823 in the model with only the control variables to 0.831 in the model distinguishing *neg_l_slope* and *r_slope*. The models are all well into Hosmer and Lemeshow's (2000) excellent range, rendering improvement in goodness of fit more difficult.

As in Panel A, the coefficients on the control variables generally are as expected or are interpretable. The coefficients on *stdret* and on the interaction of *stdret* with *loss_seq* are consistently significantly negative, reflecting debt investors' aversion to volatility. The coefficient on *loss_seq* is significantly negative in the model with only control variables in the first column, but becomes significantly positive in the models with convexity variables in the second through fourth columns. Apparently the convexity variables capture the loss sequences that are undesirable to debt investors. The coefficient on *leverage* is consistently significantly positive, consistent with levered firms having debt and tending to issue more of the same. As expected, the coefficient on *fdeficit* is consistently positive; this variable is again highly significant as in the literature on financing (Frank and Goyal 2003). The coefficient on *log_mve* is consistently significantly positive.

Consistent with hypothesis H3A, the coefficients on each of the four convexity measures are significantly negative. Consistent with hypothesis H3B, we find that the coefficient on the interaction of *loss_seq* with *neg_l_slope* is significantly negative, indicating that left-tail convexity increases the negative association of *loss_seq* with *debt_issue*. This coefficient is also much more negative than the direct coefficient on *loss_seq*, reflecting debt investors' aversion to left-tail options. While we made no hypotheses in this regard, we also find that the coefficient on the interaction of *loss_seq* with *r_slope* is significantly negative, although only 60% as large as the coefficient on the interaction of *loss_seq* with *neg_l_slope*.

In column 5 of both panels in Table 8, we tabulate the results of estimating equations [2] and [3] with convexity estimated using only the first three partitions, i.e., without the market continuation partition. This convexity is reported in the "investment real optionality" column of Table 6. We make this modification to address the concern that equity issuances are positively serially correlated, and that the calculation of convexity partitioning on current equity issuances may mechanically capture this serial correlation, although how this would occur is not apparent. We add current year equity issuance to the regression models for similar reasons. For brevity, we tabulate results only for the models with the *r_slope* and *neg_l_slope* convexity measures; the models with the other convexity measures yield similar results. In each panel, column 5 reports that the coefficients on both convexity measures and their interactions with *loss_seq* maintain the same sign as in column 4 of that panel and remain significant. The significance levels on these coefficients drop somewhat from column 4, however, consistent with the convexity estimates being noisier. While the coefficient of *stdret* remains positive in column 5 of Panel A, the magnitude and significance of the coefficient are considerably lower than in the base model reported in column 1. This suggests that the 12-path convexity measures continue to capture meaningful portions of the positive implications of volatility.

Collectively, these results provide strong support for the ability of our convexity measures to predict future financings, particularly equity issuances. [26] Equity investors embrace convexity but not volatility controlling for convexity. Debt investors eschew volatility whether or not accompanied by convexity.

[6] Conclusion

In this paper, we provide evidence that returns-earnings convexity results from right-tail continuation options, not solely from the left-tail abandonment and limited liability of equity options that are the focus of prior research. We estimate convexity using a simple spline regression/sequential partitioning approach. We employ novel graphical methods that demonstrate the goodness of fit of this simple approach. We provide evidence that our measures of convexity explain future equity and debt issuances in a fashion consistent with them capturing the positive implications of volatility from the perspective of equityholders.

Our approach suggests many avenues for future research; we mention only two. First, we provide preliminary analysis of how convexity, particularly right-tail convexity, varies with

[26] Untabulated specification analyses indicate that the relationships between our convexity measures and equity and debt financings reported in Table 8 are highly robust to the inclusion of the following additional control variables: firm age, asset tangibility (property, plant, and equipment), Altman's Z score, level of R&D expenditures, total assets, and market-to-book. The results are also robust to estimating convexity in periods prior to the prediction of equity and debt financings. Specifically, we estimate convexity for the first 31 years of our sample period (1974-2004) and then predict equity and debt issuances in the last 5 years of our sample period (2005-2009). The inferences remain unchanged.

lifecycle stage based on a modification of Dickinson's (2011) lifecycle proxy. While we find convexity varies with lifecycle stage largely as predicted, we also unexpectedly find that the vertical location of the spline knot varies considerably with lifecycle stage. We believe that the effect of lifecycle stage on the left- and right-tail options that yield returns-earnings convexity deserve research attention. Second, we provide evidence that convexity captures the positive implications of volatility from the perspective of equityholders. Most research in accounting and finance treats volatility as a negative, even from the perspective of equityholders. This view is often and perhaps usually incorrect. We believe many literatures in accounting and finance should be revisited to identify the positive implications of volatility for equityholders and other similarly positioned claimants. For example, our preliminary results in a separate project indicate that convexity is significantly negatively associated with the probability of bankruptcy controlling for other predictors of bankruptcy.

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Appendix A ■ Variable definitions

We assemble a sample of 142,071 firm-year observations for the period of 1974 through 2009. We limit our analysis to those observations where both beginning and ending year price exceed $2 per share. We winsorize all continuous firm-year observation variables at the 1% and 99% levels.

$\boldsymbol{R_t}$	Return on the firm's stock over the 12-month period commencing with the 4th month after the end of fiscal year $t - 1$. We obtain returns data from CRSP.
$\boldsymbol{X_t}$	Earnings per share before extraordinary items for year t (Compustat item 'EPSPX').
loss$_t$	An indicator variable denoting whether the firm reported a loss in year t; 0 otherwise. Specifically, we set loss equal to 1 if X_t is less than or equal to 0.
loss_seq$_t$	Number of consecutive annual losses prior to and including year t (maximum 6).
stdret$_t$	The standard deviation of daily returns over the 12-month period commencing with the 4th month after the end of fiscal year $t - 1$.
$\boldsymbol{P_t}$	Year t ending price (Compustat item 'PRCC_F').
high_rd$_t$	An indicator variable set to 1 if R&D expense for year t (Compustat item 'XRD') divided by sales for year t (Compustat item 'SALE') exceeds 5%; 0 otherwise.
equity_issue$_t$	An indicator variable set to 1 if the firm's net equity issues in year t are positive; 0 otherwise. Following Frank and Goyal (2003), we calculate net equity issue as sales of common and preferred stock (Compustat item 'SSTK') minus purchases of common and preferred stock (Compustat item 'PRSTKC').
debt_issue$_t$	An indicator variable set to 1 if the firm's net debt issue in year t is positive; 0 otherwise. Following Frank and Goyal (2003), we calculate net debt issue as long-term debt issuance (Compustat item 'DLTIS') minus long-term debt reductions (Compustat item 'DLTR').
fdeficit$_t$	The firm's financing deficit for year t, scaled by year $t - 1$ ending total assets (Compustat item 'AT'). Following Frank and Goyal (2003), we calculate the firm's financing deficit for the year as cash dividends plus investments plus change in working capital minus internal cash flow. As noted in Frank and Goyal (2003, p. 229), the calculations of investments, working capital changes, and internal cash flows differ depending up the firm's reporting format code.
mve$_t$	Market value of equity at the end of year t (P_t multiplied by Compustat item 'CSHO').
log_mve$_t$	The log of mve.
leverage$_t$	Long-term debt (Compustat item 'DLTT') at the end of year t, divided by mve.
neg_l_slope	The negative of the returns-earnings spline regression slope to the left of the spline knot.
r_slope	The returns-earnings spline regression slope to the right of the spline knot.
rl_slope_diff	The difference between right regime and left regime slopes from the spline regression.
ols_spl_dist	The vertical distance between the OLS regression fitted value and the spline regression fitted value at the knot point.
SD(*X*)	The standard deviation of X_t/P_{t-1}.
mod_LC$_t$	The year t cash-flow-statement-classification-based proxy for lifecycle stage developed by Dickinson (2011), modified to reclassify R&D outlays as investing as opposed to operating cash flows. Refer to Appendix B for details.

Appendix B ■ Modified Dickinson (2011) lifecycle measure

We identify lifecycle stage using a modification of Dickinson's (2011) cash flow statement-classification-based proxy. As noted in her footnote 7, Dickinson classifies observations into introduction, growth, mature, shake-out and decline stages based on the signs of their operating (OCF), investing (ICF), and financing (FCF) cash flows. Specifically, she classifies the eight potential combinations of positive and negative cash flow activity patterns as follows:

Predicted sign	[1] Introduction	[2] Growth	[3] Mature	[4] Shake-out			[5] Decline
OCF	−	+	+	−	+	+	−
ICF	−	−	−	−	+	+	+
FCF	+	+	−	−	+	−	−

We modify her approach in order to better capture investments in real options, which is the focus of this paper. In particular, we classify research and development expenditures (R&D) as investing rather than operating cash outflows. This yields modified operating cash flows (OCF^M) and modified investing cash flows (ICF^M), as follows:

$OCF^M = OCF + R\&D.$

$ICF^M = ICF - R\&D.$

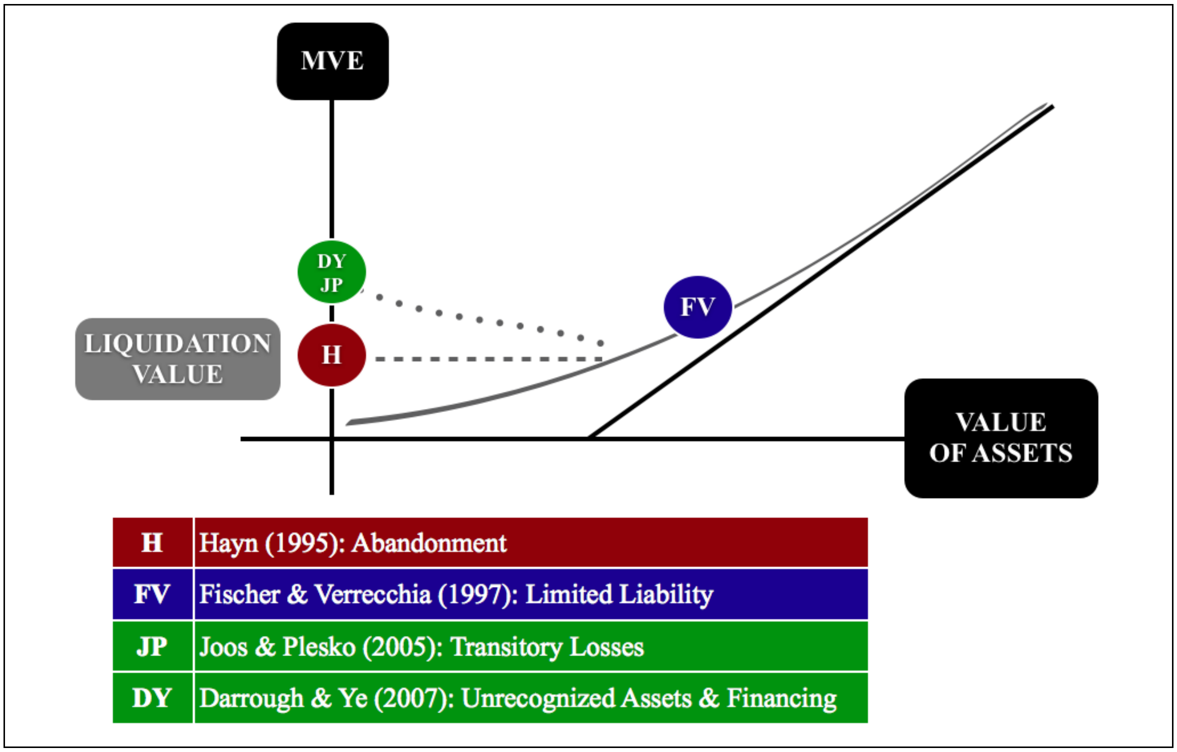
From this modification, we obtain a modifed lifecycle measure, *mod_LC*. The effects of this modification are to move observations: (1) from the introduction stage to the growth stage (because of the increase in OCF), (2) from the shake-out stage to the growth and mature stages (because of the decrease in ICF), and (3) from the decline stage into all four earlier stages (for both reasons). We illustrate the effects of this modification on the subsample of high volatility tercile firms below. The total sample of 31,495 observations is less than the high volatility tercile sample of n = 47,357 because we require cash flow statement data, which are available only after the passage of FAS 95 (ASC 230) in 1987.

		Adjusted lifecycle (*mod_LC*)					
		Introduction	Growth	Mature	Shake-out	Decline	
Original	Introduction	4,322	1,976	0	0	0	6,298
	Growth	0	9,586	0	0	0	9,586
	Mature	0	0	9,024	0	0	9,024
	Shake-out	0	258	776	2,231	0	3,265
	Decline	506	737	227	303	1,549	3,322
		4,828	12,557	10,027	2,534	1,549	31,495

Figure 1 ■ Non-linearity in the returns-earnings relation

Figure 1 summarizes theories that characterize the non-linear relation between returns (in undifferenced form as the market value of equity) and earnings (in undifferenced form as the assets in place). Value of assets should be interpreted as the economic value of assets for Fischer and Verrecchia (1997) and as the book value of assets for the other papers.

Panel A: Investors' attenuated response to losses



Panel B: Investors' heightened response to gains

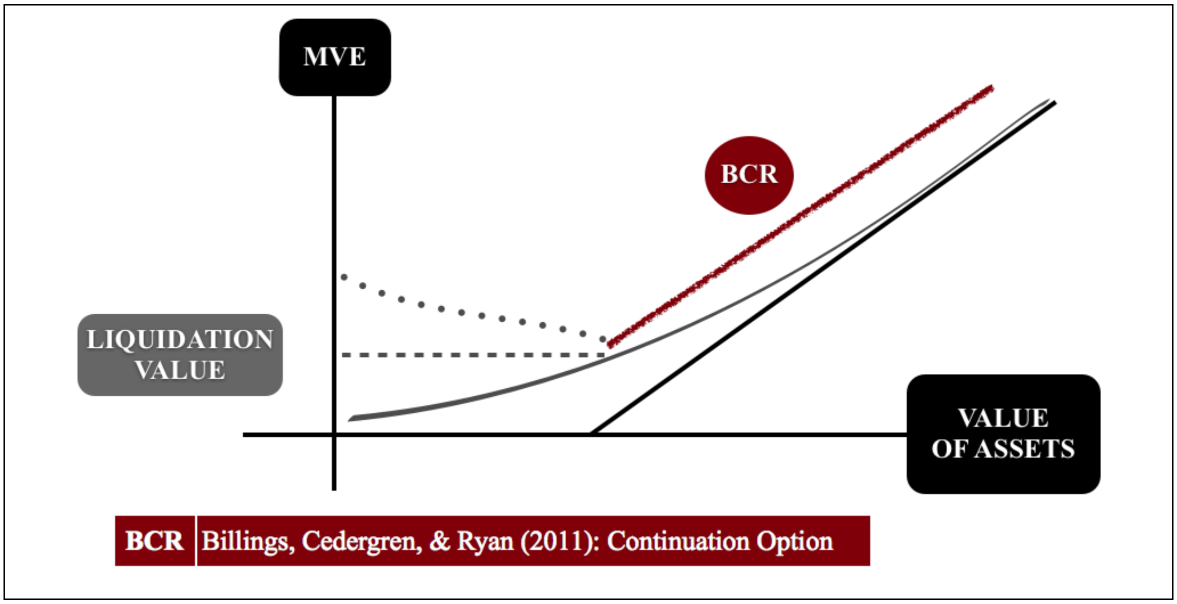


Figure 2 ■ Loss frequency by volatility and price

Figure 2 plots loss frequency by intersection of price decile and volatility decile for the full sample (n = 142,071).

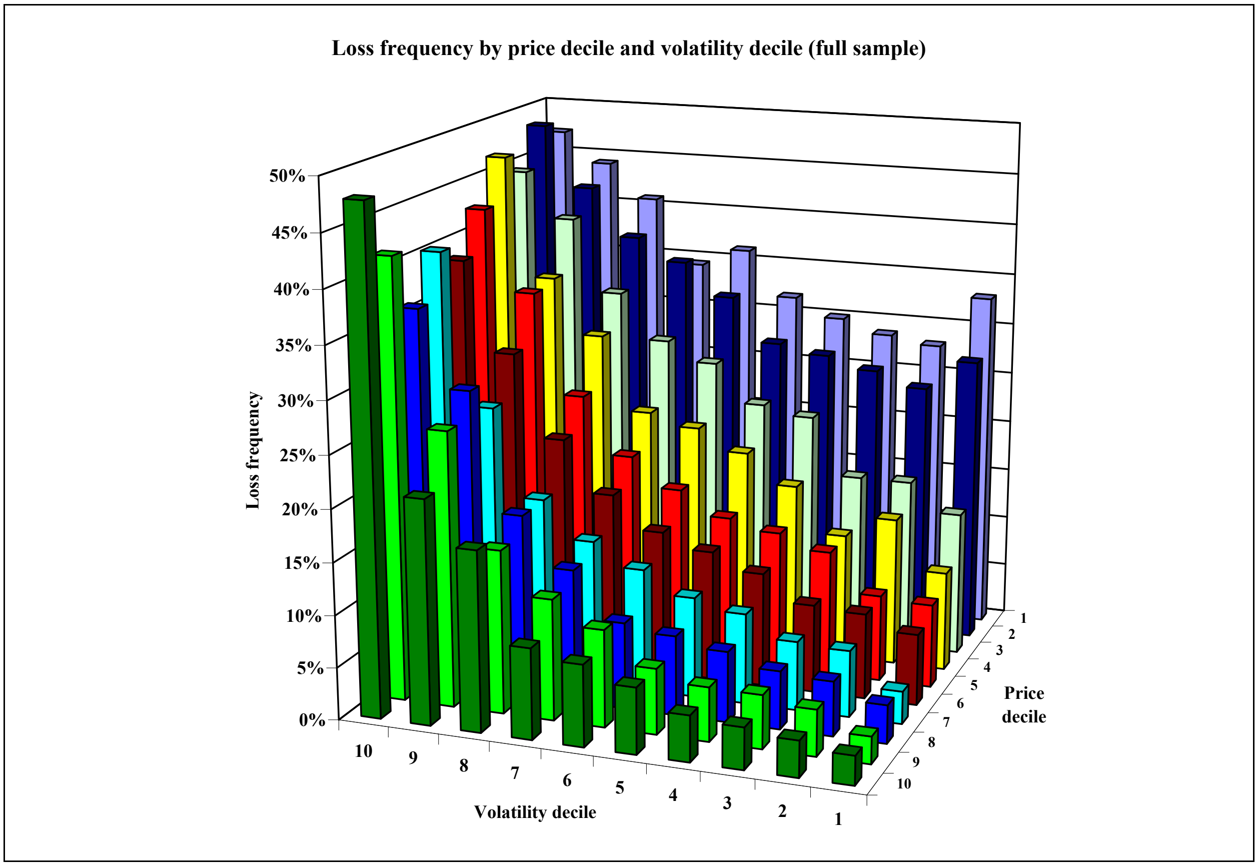


Figure 3 ■ The returns-earnings relation, full sample

The relation between returns and earnings for the full sample of firm-year observations over the period 1974 through 2009 (n = 142,071). Following Hayn (1995), we measure stock return (R_t) over the 12 months commencing with the fourth month of fiscal year t and earnings (X_t) as earnings before extraordinary items per share in year t. We scale X_t by price at the end of fiscal year $t-1$ (P_{t-1}). The blue solid line represents the fitted maximum likelihood estimation of equation [1-SPL]:

$$R_t = \alpha_{\text{SPL}} + \beta_{\text{L}} \left(\frac{X_t}{P_{t-1}} \right) + \beta_{\text{IR}} \max\left(\frac{X_t}{P_{t-1}} - x^*,\ 0 \right) + \varepsilon_t .$$

The blue fine-dashed line represents the fitted maximum likelihood estimation equation of a spline regression with two knots:

$$R_t = \alpha + \beta_1 \left(\frac{X_t}{P_{t-1}} \right) + \beta_2 \max\left(\frac{X_t}{P_{t-1}} - x^*,\ 0 \right) + \beta_3 \max\left(\frac{X_t}{P_{t-1}} - x^{**},\ 0 \right) + \varepsilon_t .$$

The green solid line represents the fitted OLS estimation in equation [1-OLS]:

$$R_t = \alpha_{\text{OLS}} + \beta_{\text{OLS}} \left(\frac{X_t}{P_{t-1}} \right) + \varepsilon_t .$$

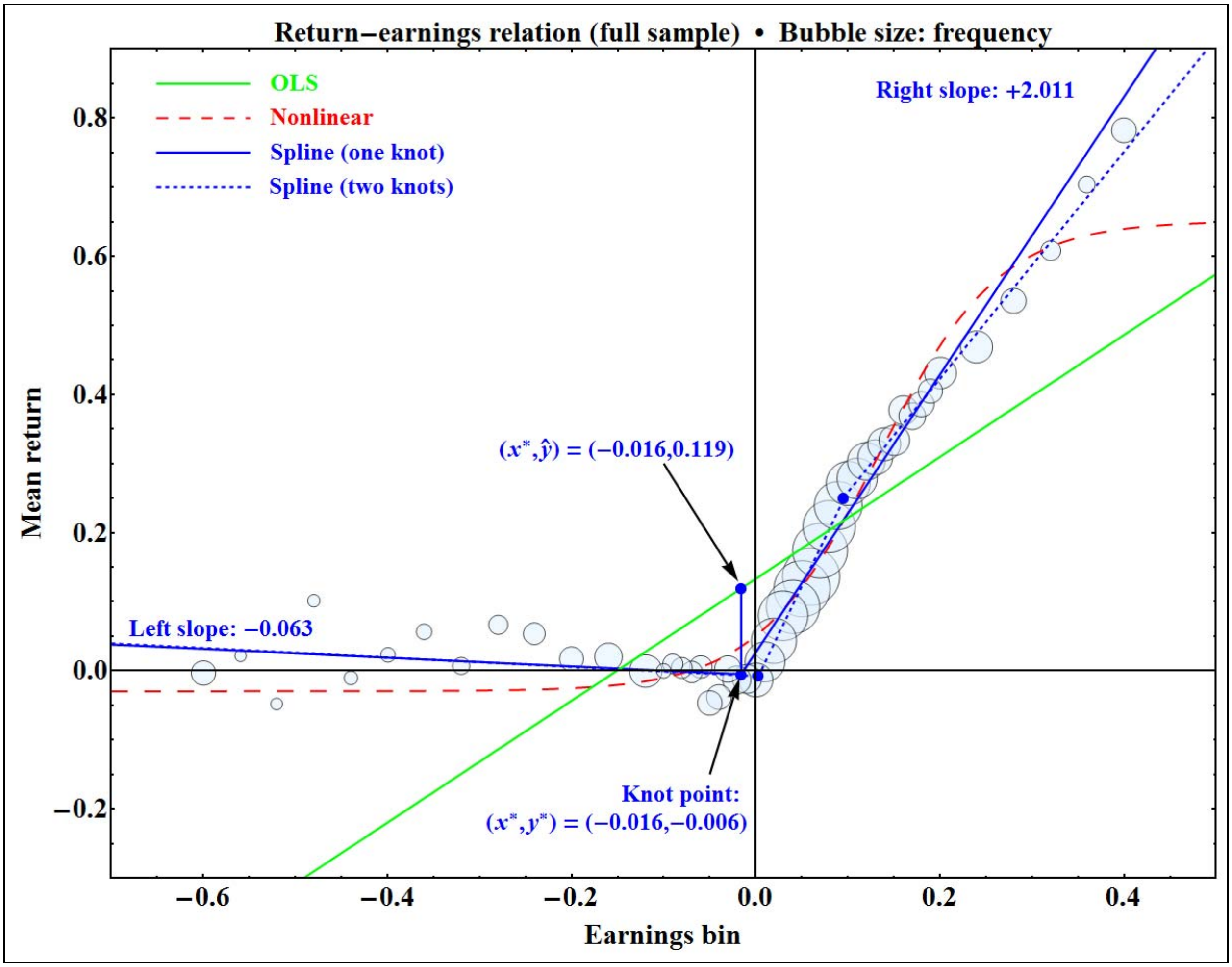
The red coarse-dashed curve represents the fitted line from the following nonlinear equation:

$$R_t = \beta_0 + \frac{\beta_1}{1 + \exp\left[\beta_2 \left(\frac{X_t}{P_{t-1}} \right) + \beta_3 \right]} .$$

As described in the notes to Table 3, we compute one-tail contributions to convexity equal to the slope to the right of the spline knot $(r\_slope = \hat{\beta}_{\text{R}} = \hat{\beta}_{\text{L}} + \hat{\beta}_{\text{IR}})$ and the negative of the slope to the left of the spline knot $(neg\_l\_slope = -\hat{\beta}_{\text{L}})$. We compute two-tail measures of convexity equal to the difference in the right and left slopes $(rl\_slope\_diff = \hat{\beta}_{\text{R}} - \hat{\beta}_{\text{L}})$ and the vertical distance between the OLS fitted value and spline fitted value at the estimated spline knot $[ols\_spl\_dist = (\hat{\alpha}_{\text{OLS}} - \hat{\alpha}_{\text{SPL}}) + (\hat{\beta}_{\text{OLS}} - \hat{\beta}_{\text{L}})x^*]$. The vertical solid line segment depicts *ols_spl_dist*. The table below the figure summarizes these convexity measures.

We place firm-year observations into bins of width 0.01 for values of deflated EPS between −0.10 and +0.20 and of width 0.04 for extreme values of deflated EPS. The vertical (horizontal) location of each bubble depicts the mean of R_t (X_t/P_{t-1}) for the bin. The size of each bubble depicts the number of observations in the bin.

Figure 3 ■ The returns-earnings relation, full sample (continued)



	Two-tail measures of convexity		One-tail contributions to convexity	
	rl_slope_diff	*ols_spl_dist*	*neg_l_slope*	*r_slope*
Full sample	**+2.074** ***	**+0.125** ***	**+0.063** **	**+2.011** ***

Figure 4 ■ The returns-earnings relation, partitioned

The relation between returns and earnings, using firm-year observations over the period 1974 through 2009. Following Hayn (1995), we measure stock return (R_t) over the 12-month period commencing with the fourth month of fiscal year t and earnings (X_t) as earnings before extraordinary items per share in year t. We scale X_t by the price at the end of year $t-1$. We place each firm-year observation into an earnings bin. The overlaying solid lines represent the fitted maximum likelihood estimation of equation [1-SPL]:

$$R_t = \alpha_{\mathrm{SPL}} + \beta_{\mathrm{L}}\left(\frac{X_t}{P_{t-1}}\right) + \beta_{\mathrm{IR}} \max\left(\frac{X_t}{P_{t-1}} - x^*,\ 0\right) + \varepsilon_t .$$

As described in the notes to Table 5, we compute one-tail contributions to convexity equal to the slope to the right of the spline knot $(r\_slope = \hat{\beta}_{\mathrm{R}} = \hat{\beta}_{\mathrm{L}} + \hat{\beta}_{\mathrm{IR}})$ and the negative of the slope to the left of the spline knot $(neg\_l\_slope = -\hat{\beta}_{\mathrm{L}})$. We compute two-tail measures of convexity equal to the difference in the right and left slopes $(rl\_slope\_diff = \hat{\beta}_{\mathrm{R}} - \hat{\beta}_{\mathrm{L}})$ and the vertical distance between the OLS fitted value and spline fitted value at the estimated spline knot $[ols\_spl\_dist = (\hat{\alpha}_{\mathrm{OLS}} - \hat{\alpha}_{\mathrm{SPL}}) + (\hat{\beta}_{\mathrm{OLS}} - \hat{\beta}_{\mathrm{L}})x^*]$. The vertical solid line segment depicts *ols_spl_dist*. The table below each figure summarizes these convexity measures.

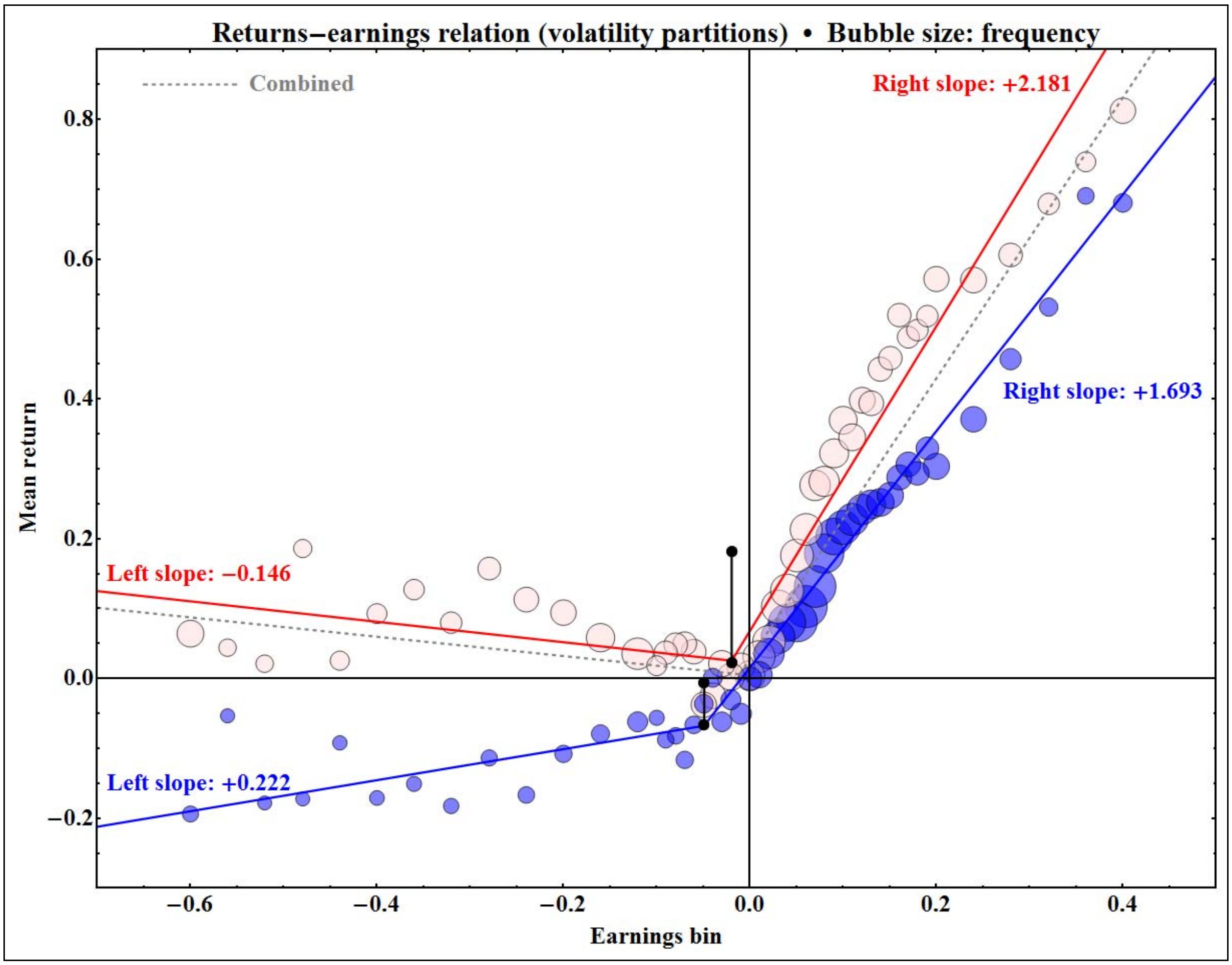
Panels A through D split the full sample into subsamples, sequentially partitioned on the four dimensions described in the optionality tree provided in Table 4, Panel A. In Panels A through C2, we place firm-year observations into bins of width 0.01 for values of deflated EPS between −0.10 and +0.20 and of width 0.04 for extreme values of deflated EPS. In Panel D, we place firm-year observations into bins of width 0.02 for values of deflated EPS between −0.10 and +0.20 and of width 0.08 for extreme values of deflated EPS. The size of each bubble depicts the number of observations in the bin. The dashed lines represent the results of the spline regression when combining the data from both subsamples.

Figure 4 ■ The returns-earnings relation, partitioned (continued)

Panel A: Partitioned based on volatility

RED = High volatility
(paths 1 – 8, *n* = 47,357)

BLUE = Low volatility
(paths 17 – 24, *n* = 47,346)



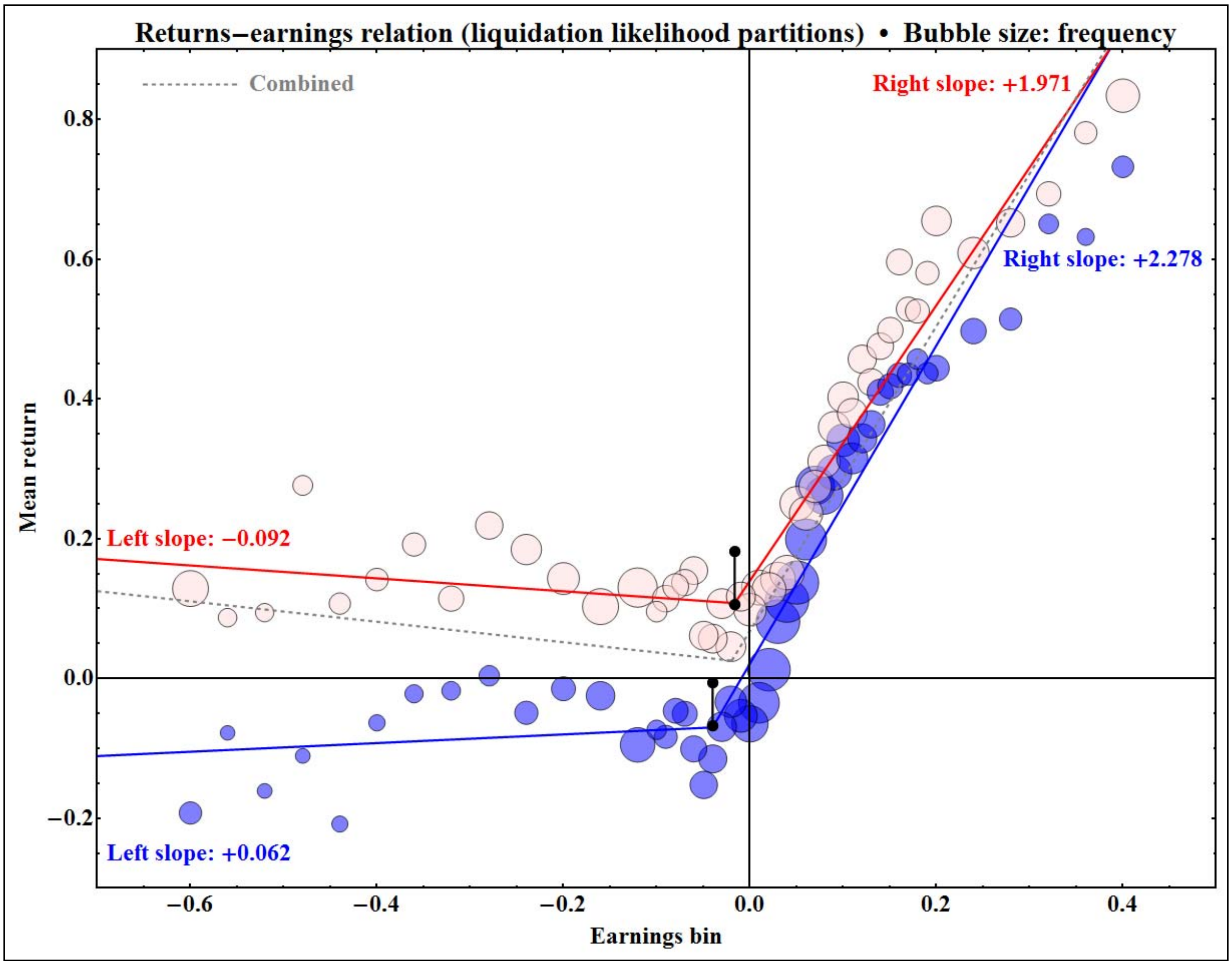
	Two-tail measures of convexity		One-tail contributions to convexity	
	rl_slope_diff	*ols_spl_dist*	*neg_l_slope*	*r_slope*
High volatility	+2.327	+0.159	+0.146	+2.181
Low volatility	+1.471	+0.050	−0.222	+1.693
High − Low	**+0.856** ***	**+0.109** ***	**+0.368** ***	**+0.488** ***

Figure 4 ■ The returns-earnings relation, partitioned (continued)

Panel B: High volatility firms, partitioned based on liquidation likelihood

RED ◯ = High volatility, high liquidation likelihood
(paths 5 – 8, *n* = 23,515)

BLUE ● = High volatility, low liquidation likelihood
(paths 1 – 4, *n* = 23,842)



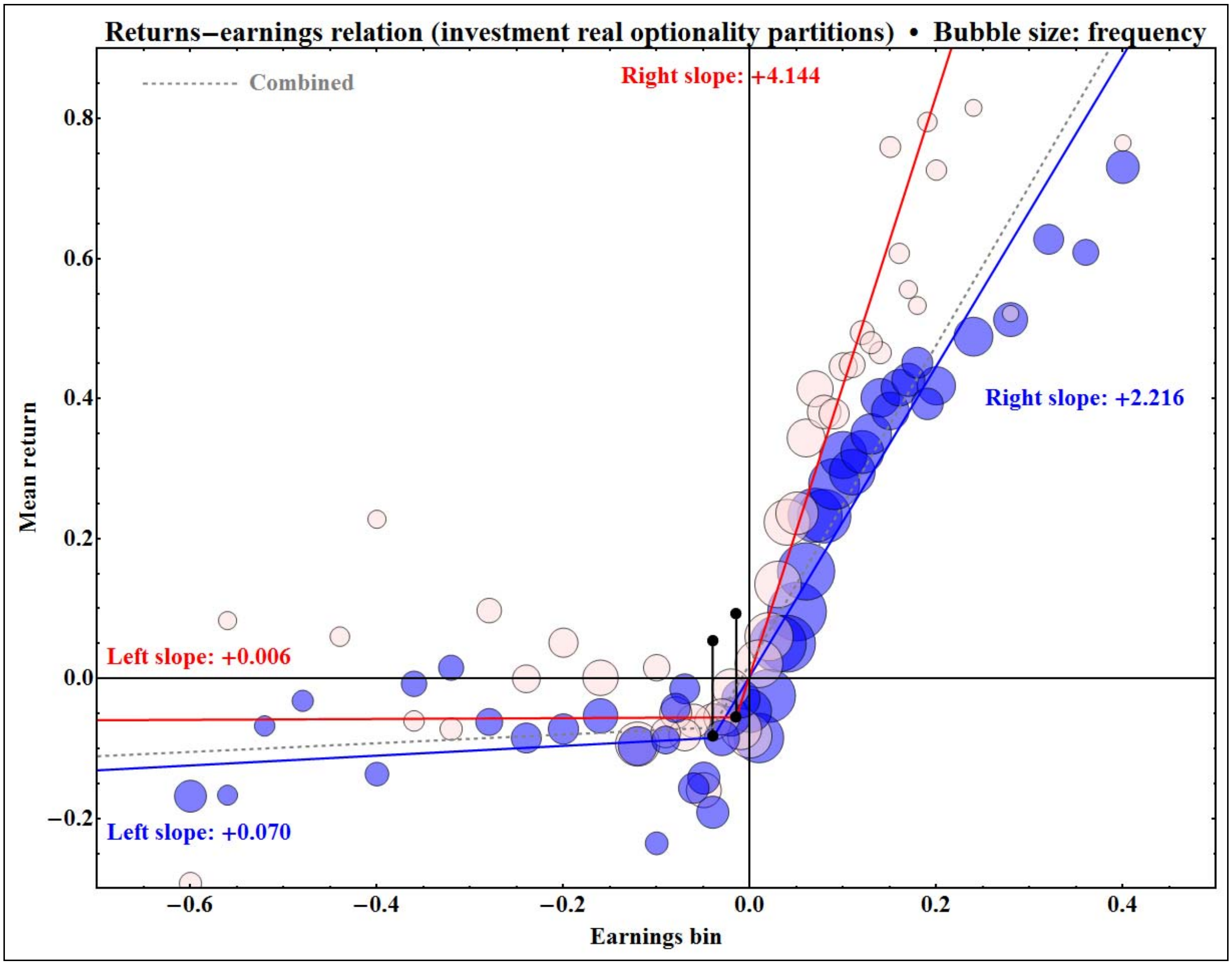
	Two-tail measures of convexity		One-tail contributions to convexity	
	rl_slope_diff	*ols_spl_dist*	*neg_l_slope*	*r_slope*
High liquidation likelihood	+2.063	+0.162	+0.092	+1.971
Low liquidation likelihood	+2.216	+0.125	−0.062	+2.278
High − Low	**−0.153**	**+0.037** ***	**+0.154** *	**−0.307** ***

Figure 4 ■ The returns-earnings relation, partitioned (continued)

Panel C1: High volatility and low liquidation likelihood observations, partitioned based on investment real optionality

RED = High volatility, low liquidation likelihood, high investment real optionality (paths 1 – 2, *n* = 7,872)

BLUE = High volatility, low liquidation likelihood, low investment real optionality (paths 3 – 4, *n* = 15,643)



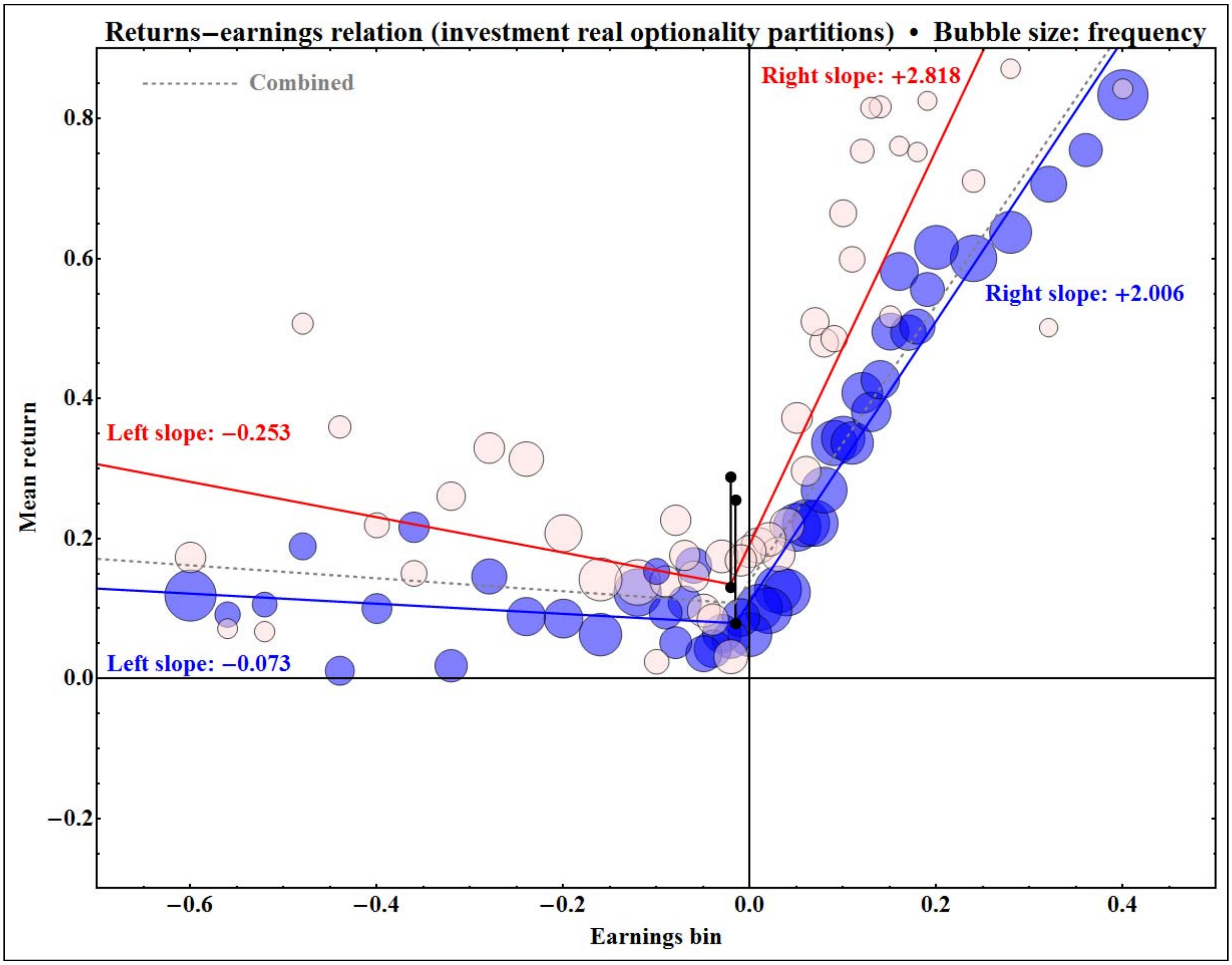
	Two-tail measures of convexity		One-tail contributions to convexity	
	rl_slope_diff	*ols_spl_dist*	*neg_l_slope*	*r_slope*
High investment real optionality	+4.138	+0.148	−0.006	+4.144
Low investment real optionality	+2.146	+0.136	−0.070	+2.216
High − Low	**+1.992** ***	**+0.012** ***	**+0.064** ***	**+1.928** ***

Figure 4 ■ The returns-earnings relation, partitioned (continued)

Panel C2: High volatility and high liquidation likelihood observations, partitioned based on investment real optionality

RED ○ = High volatility, high liquidation likelihood, high investment real optionality (paths 5 – 6, *n* = 6,394)

BLUE ● = High volatility, high liquidation likelihood, low investment real optionality (paths 7 – 8, *n* = 17,448)



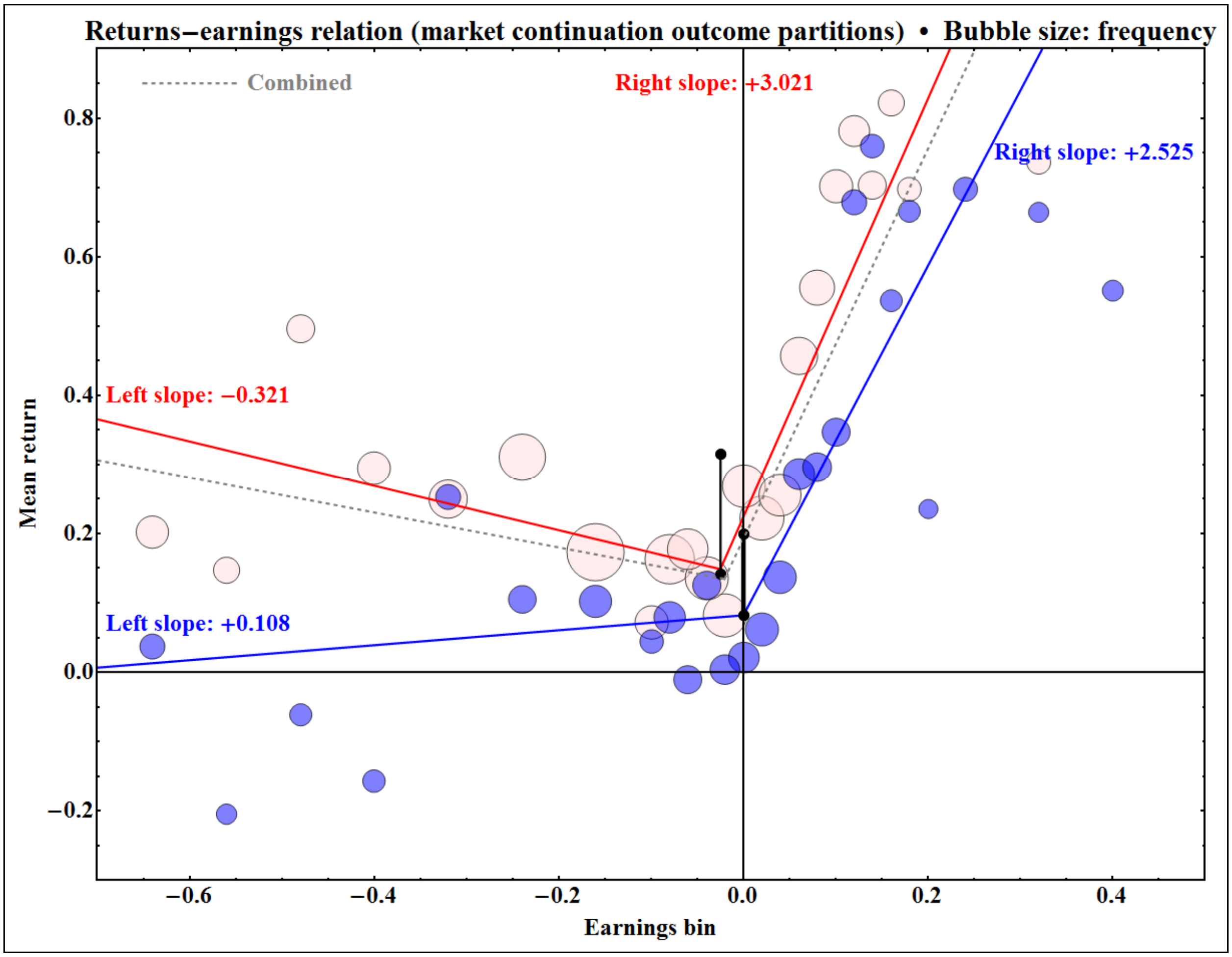
	Two-tail measures of convexity		One-tail contributions to convexity	
	rl_slope_diff	*ols_spl_dist*	*neg_l_slope*	*r_slope*
High investment real optionality	+3.071	+0.159	+0.253	+2.818
Low investment real optionality	+2.079	+0.177	+0.073	+2.006
High − Low	**+0.992** ***	**−0.018** ***	**+0.180** *	**+0.812** ***

Figure 4 ■ The returns-earnings relation, partitioned (continued)

Panel D: High volatility, high liquidation likelihood, and high investment real optionality observations, partitioned based on market continuation outcome

RED = High volatility, high liquidation likelihood, high investment real optionality, positive market continuation (path 5, *n* = 5,048)

BLUE = High volatility, high liquidation likelihood, high investment real optionality, negative market continuation (path 6, *n* = 1,346)



	Two-tail measures of convexity		One-tail contributions to convexity	
	rl_slope_diff	*ols_spl_dist*	*neg_l_slope*	*r_slope*
Positive continuation	+3.342	+0.173	+0.321	+3.021
Negative continuation	+2.417	+0.117	−0.108	+2.525
Positive − Negative	**+0.925** *	**+0.056** ***	**+0.429** **	**+0.496**

Table 1 ■ Descriptive statistics

The sample consists of 142,071 firm-year observations from 1974–2009. Panel A (B) partitions the sample by profit and loss (volatility tercile). In both panels, ***, **, * denote instances where the two subsamples differ significantly at the 1%, 5%, and 10% level, respectively, for two-tailed tests. Please refer to Appendix A for variable definitions.

Panel A: Full sample, partitioned by profit and loss

	Profit firm-years (n = 113,639)			Loss firm-years (n = 28,432)			Differences (Profit − Loss)		
	Mean	Median	St. dev.	Mean	Median	St. dev.	Mean	Median	St. dev.
$\boldsymbol{R_t}$	0.216	0.135	0.500	0.003	−0.141	0.637	0.213 ***	0.276 ***	−0.137 ***
$\boldsymbol{X_t/P_{t-1}}$	0.094	0.074	0.075	−0.148	−0.085	0.164	0.242 ***	0.159 ***	−0.089 ***
loss$_t$	0.000	0.000	0.000	1.000	1.000	0.000	n/a	n/a	n/a
loss_seq$_t$	0.000	0.000	0.000	2.299	2.000	1.593	n/a	n/a	n/a
stdret$_t$	0.027	0.024	0.014	0.040	0.037	0.020	−0.013 ***	−0.013 ***	−0.006 ***
$\boldsymbol{P_{t-1}}$	22.17	17.70	17.39	12.17	8.13	12.18	10.00 ***	9.58 ***	5.21 ***
high_rd$_t$	0.118	0.000	0.323	0.336	0.000	0.472	−0.218 ***	0.000 ***	−0.149 ***
equity_issue$_t$	0.464	0.000	0.499	0.614	1.000	0.487	−0.150 ***	−1.000 ***	0.012 ***
debt_issue$_t$	0.363	0.000	0.481	0.344	0.000	0.475	0.019 ***	0.000 ***	0.006 **
fdeficit$_t$	0.021	0.001	0.101	0.080	0.011	0.190	−0.059 ***	−0.010 ***	−0.089 ***
leverage$_t$	0.495	0.216	0.781	0.634	0.163	1.090	−0.139 ***	0.053 ***	−0.309 ***
log_mve$_t$	5.447	5.314	2.133	4.587	4.439	1.827	0.860 ***	0.875 ***	0.306 ***

Panel B: Full sample partitioned based on high volatility tercile and low volatility tercile

	High volatility (n = 47,357)			Low volatility (n = 47,346)			Differences (High − Low)		
	Mean	Median	St. dev.	Mean	Median	St. dev.	Mean	Median	St. dev.
$\boldsymbol{R_t}$	0.201	0.043	0.695	0.153	0.123	0.351	0.048 ***	−0.080 ***	0.344 ***
$\boldsymbol{X_t/P_{t-1}}$	0.005	0.031	0.179	0.077	0.072	0.086	−0.072 ***	−0.041 ***	0.093 ***
loss$_t$	0.367	0.000	0.482	0.071	0.000	0.257	0.296 ***	0.000 ***	0.225 ***
loss_seq$_t$	0.946	0.000	1.603	0.320	0.000	0.926	0.626 ***	0.000 ***	0.677 ***
stdret$_t$	0.046	0.042	0.016	0.016	0.016	0.005	0.030 ***	0.026 ***	0.011 ***
$\boldsymbol{P_{t-1}}$	11.08	7.38	11.28	29.77	25.50	18.53	−18.69 ***	−18.13 ***	−7.25 ***
high_rd$_t$	0.301	0.000	0.459	0.054	0.000	0.226	0.247 ***	0.000 ***	0.233 ***
equity_issue$_t$	0.610	1.000	0.488	0.379	0.000	0.485	0.231 ***	1.000 ***	0.003 ***
debt_issue$_t$	0.314	0.000	0.464	0.397	0.000	0.489	−0.083 ***	0.000 ***	−0.025 ***
fdeficit$_t$	0.059	0.006	0.165	0.015	0.000	0.085	0.044 ***	0.006 ***	0.080 ***
leverage$_t$	0.533	0.109	1.007	0.528	0.289	0.715	0.005	−0.180 ***	0.292 ***
log_mve$_t$	6.226	6.248	2.161	4.275	4.081	1.725	1.951 ***	2.167 ***	0.436 ***

Table 2 ■ Correlations

Table 2 presents correlations for the full sample (n = 142,071). We present Pearson (Spearman) correlations below (above) the diagonal. Correlations in boldface are significant at the 1% level. Please refer to Appendix A for variable definitions.

	R_t	X_t/P_{t-1}	$loss_t$	$loss\_seq_t$	$stdret_t$	P_{t-1}	$high\_rd_t$	$equity\_issue_t$	$debt\_issue_t$	$fdeficit_t$	$leverage_t$	log_mve_t
R_t		**0.3543**	**−0.2303**	**−0.2212**	**−0.0469**	**−0.0528**	**−0.0641**	−0.0041	**−0.0525**	**−0.0437**	**0.0766**	**0.0930**
X_t/P_{t-1}	**0.2265**		**−0.6930**	**−0.6897**	**−0.2773**	**0.0762**	**−0.2910**	**−0.1575**	−0.0017	**−0.1173**	**0.1896**	**−0.0394**
$loss_t$	**−0.1583**	**−0.6983**		**0.9926**	**0.3317**	**−0.2984**	**0.2367**	**0.1203**	**−0.0156**	**0.1046**	**−0.0316**	**−0.1629**
$loss\_seq_t$	**−0.0880**	**−0.5755**	**0.7905**		**0.3420**	**−0.3079**	**0.2525**	**0.1296**	**−0.0205**	**0.1148**	**−0.0404**	**−0.1596**
$stdret_t$	**0.0762**	**−0.2652**	**0.3386**	**0.3423**		**−0.5544**	**0.2913**	**0.1811**	**−0.0998**	**0.0638**	**−0.1641**	**−0.3284**
P_{t-1}	**−0.0965**	**0.0899**	**−0.2360**	**−0.2177**	**−0.3912**		**−0.0818**	**−0.0439**	**0.0906**	**0.0078**	**−0.0734**	**0.6874**
$high\_rd_t$	**−0.0097**	**−0.2066**	**0.2367**	**0.2788**	**0.2788**	**−0.0398**		**0.1975**	**−0.0950**	**0.0952**	**−0.3554**	**0.0403**
$equity\_issue_t$	**0.0332**	**−0.1116**	**0.1203**	**0.1480**	**0.1709**	**−0.0610**	**0.1975**		**0.0816**	**0.3658**	**−0.0714**	**0.0741**
$debt\_issue_t$	**−0.0536**	0.0007	**−0.0156**	**−0.0334**	**−0.0975**	**0.0823**	**−0.0950**	**0.0816**		**0.6094**	**0.1698**	**0.0802**
$fdeficit_t$	−0.0069	**−0.1509**	**0.1921**	**0.2634**	**0.1560**	**−0.0623**	**0.1542**	**0.2994**	**0.3660**		**−0.0747**	**0.0214**
$leverage_t$	**0.0726**	**−0.0275**	**0.0652**	**0.0368**	**0.0076**	**−0.1650**	**−0.2154**	**−0.0722**	**0.0605**	**−0.0948**		**−0.0461**
log_mve_t	**0.0588**	**0.0267**	**−0.1634**	**−0.1142**	**−0.2816**	**0.6525**	**0.0449**	**0.0564**	**0.0855**	**−0.0213**	**−0.1193**	

Table 3 ■ Convexity and the returns-earnings relation, full sample

The relation between returns and earnings for the full sample of firm-year observations over the period 1974 through 2009 (n = 142,071). Following Hayn (1995), we measure stock return (R_t) over the 12 months commencing with the fourth month of fiscal year t and earnings (X_t) as earnings before extraordinary items per share in year t. We scale X_t by price at the end of fiscal year $t-1$ (P_{t-1}). We regress R_t on X_t/P_{t-1} using a spline regression with a single knot located at $X_t/P_{t-1} = x^*$. The spline regression is given by equation [1-SPL]:

$$R_t = \alpha_{\text{SPL}} + \beta_{\text{L}}\left(\frac{X_t}{P_{t-1}}\right) + \beta_{\text{IR}} \max\left(\frac{X_t}{P_{t-1}} - x^*,\ 0\right) + \varepsilon_t .$$

Additionally, we also estimate the analogous OLS regression equation [1-OLS]:

$$R_t = \alpha_{\text{OLS}} + \beta_{\text{OLS}}\left(\frac{X_t}{P_{t-1}}\right) + \varepsilon_t .$$

We compute one-tail contributions to convexity equal to the slope to the right of the spline knot ($r\_slope = \hat{\beta}_{\text{R}} = \hat{\beta}_{\text{L}} + \hat{\beta}_{\text{IR}}$) and the negative of the slope to the left of the spline knot ($neg\_l\_slope = -\hat{\beta}_{\text{L}}$). We compute two-tail measures of convexity equal to the difference in the right and left slopes ($rl\_slope\_diff = \hat{\beta}_{\text{R}} - \hat{\beta}_{\text{L}}$) and the vertical distance between the OLS fitted value and spline fitted value at the estimated spline knot ($ols\_spl\_dist = \hat{y} - y^*$). The symbols ***, **, and * denote significance at the 1%, 5%, and 10% levels for two-tailed tests, respectively. We compute t-statistics using standard errors clustered by firm and year.

Spline regression: Left regime slope (β_{L}) (n = 24,668)	Spline regression: Right regime slope (β_{R}) (n = 117,403)	Test of differences (Right − Left)
−0.063 **	**2.011** ***	**2.074** ***
−2.27	85.65	57.22

Spline knot: x^*	Spline knot: y^*	OLS regression: α_{OLS}	OLS regression: β_{OLS}	$\hat{y} = \alpha + \beta x^*$	$\hat{y} - y^*$
−0.016	−0.006	0.133 ***	0.883 ***	0.119	**0.125** ***
		81.74	61.99		802.47

	Two-tail measures of convexity: *rl_slope_diff*	Two-tail measures of convexity: *ols_spl_dist*	One-tail contributions to convexity: *neg_l_slope*	One-tail contributions to convexity: *r_slope*
Full sample	**+2.074**	**+0.125**	**+0.063**	**+2.011**
	***	***	**	***

Table 4 ■ Optionality tree

Panel A: Frequency and loss percentage by path

Panel A outlines the four key partitions of the full sample, resulting in 24 unique paths. The high, middle, and low volatility groups represent the highest, middle, and lowest terciles of volatility in the sample, respectively. The low (high) liquidation likelihood group represents firm-year observations above (below) the median price within that volatility group. The high (low) investment real optionality group represents firm-year observations where R&D exceeds (falls below) 5% of sales. The yes (no) market continuation group represents firm-year observations where net equity issuance exceeds (falls below) zero. We list the number of firm-year observations and associated loss frequency within each cell. Please refer to Appendix A for variable definitions. Panel B provides convexity measures when partitioning the sample by the four branches independently.

<table>
<tr><th>VOLATILITY</th><th>LIQUIDATION LIKELIHOOD</th><th>INVESTMENT REAL OPTIONALITY</th><th>MARKET CONTINUATION</th><th rowspan="2">PATH</th></tr>
<tr><th>stdret</th><th>inverse price</th><th>high_rd</th><th>equity_issue</th></tr>
<tr><td rowspan="8">High
[n = 47,357 | 37% Loss]</td><td rowspan="4">Low
[n = 23,515 | 30% Loss]</td><td rowspan="2">High
[n = 7,872 | 45% Loss]</td><td>Yes [n = 6,436 | 48% Loss]</td><td>(1)</td></tr>
<tr><td>No [n = 1,436 | 33% Loss]</td><td>(2)</td></tr>
<tr><td rowspan="2">Low
[n = 15,643 | 22% Loss]</td><td>Yes [n = 8,959 | 24% Loss]</td><td>(3)</td></tr>
<tr><td>No [n = 6,684 | 19% Loss]</td><td>(4)</td></tr>
<tr><td rowspan="4">High
[n = 23,842 | 44% Loss]</td><td rowspan="2">High
[n = 6,394 | 64% Loss]</td><td>Yes [n = 5,048 | 67% Loss]</td><td>(5)</td></tr>
<tr><td>No [n = 1,346 | 50% Loss]</td><td>(6)</td></tr>
<tr><td rowspan="2">Low
[n = 17,448 | 36% Loss]</td><td>Yes [n = 8,457 | 39% Loss]</td><td>(7)</td></tr>
<tr><td>No [n = 8,991 | 33% Loss]</td><td>(8)</td></tr>
<tr><td rowspan="8">Middle
[n = 47,368 | 16% Loss]</td><td rowspan="4">Low
[n = 23,607 | 10% Loss]</td><td rowspan="2">High
[n = 3,336 | 16% Loss]</td><td>Yes [n = 2,126 | 19% Loss]</td><td>(9)</td></tr>
<tr><td>No [n = 1,210 | 11% Loss]</td><td>(10)</td></tr>
<tr><td rowspan="2">Low
[n = 20,271 | 9% Loss]</td><td>Yes [n = 10,199 | 10% Loss]</td><td>(11)</td></tr>
<tr><td>No [n = 10,072 | 7% Loss]</td><td>(12)</td></tr>
<tr><td rowspan="4">High
[n = 23,761 | 23% Loss]</td><td rowspan="2">High
[n = 2,793 | 38% Loss]</td><td>Yes [n = 1,829 | 41% Loss]</td><td>(13)</td></tr>
<tr><td>No [n = 964 | 32% Loss]</td><td>(14)</td></tr>
<tr><td rowspan="2">Low
[n = 20,968 | 21% Loss]</td><td>Yes [n = 9,186 | 22% Loss]</td><td>(15)</td></tr>
<tr><td>No [n = 11,782 | 20% Loss]</td><td>(16)</td></tr>
<tr><td rowspan="8">Low
[n = 47,346 | 7% Loss]</td><td rowspan="4">Low
[n = 23,625 | 4% Loss]</td><td rowspan="2">High
[n = 1,672 | 6% Loss]</td><td>Yes [n = 660 | 8% Loss]</td><td>(17)</td></tr>
<tr><td>No [n = 1,012 | 4% Loss]</td><td>(18)</td></tr>
<tr><td rowspan="2">Low
[n = 21,953 | 4% Loss]</td><td>Yes [n = 8,133 | 4% Loss]</td><td>(19)</td></tr>
<tr><td>No [n = 13,820 | 3% Loss]</td><td>(20)</td></tr>
<tr><td rowspan="4">High
[n = 23,721 | 10% Loss]</td><td rowspan="2">High
[n = 881 | 24% Loss]</td><td>Yes [n = 398 | 27% Loss]</td><td>(21)</td></tr>
<tr><td>No [n = 483 | 22% Loss]</td><td>(22)</td></tr>
<tr><td rowspan="2">Low
[n = 22,840 | 10% Loss]</td><td>Yes [n = 8,745 | 9% Loss]</td><td>(23)</td></tr>
<tr><td>No [n = 14,095 | 10% Loss]</td><td>(24)</td></tr>
</table>

Table 4 ■ Optionality tree (continued)

Panel B: Convexity measures for each partition considered independently

		Two-tail measures of convexity		One-tail contributions to convexity	
	n	*rl_slope_diff*	*ols_spl_dist*	*neg_l_slope*	*r_slope*
All high volatility firm-years	47,357	+2.327	+0.159	+0.146	+2.181
All low volatility firm-years	47,346	+1.471	+0.050	−0.222	+1.693
High − Low		+0.856 ***	+0.109 ***	+0.368 ***	+0.488 ***
All high liquidation likelihood firm-years	70,747	+2.081	+0.135	+0.040	+2.041
All low liquidation likelihood firm-years	71,324	+1.809	+0.088	−0.110	+1.919
High − Low		+0.272 ***	+0.047 ***	+0.150 **	+0.122 **
All high investment real optionality firm-years	22,948	+3.750	+0.160	+0.320	+3.430
All low investment real optionality firm-years	119,123	+2.045	+0.128	+0.023	+2.022
High − Low		+1.705 ***	+0.032 ***	+0.297 ***	+1.408 ***
All positive market continuation firm-years	70,176	+2.434	+0.143	+0.164	+2.270
All negative market continuation firm-years	71,895	+1.936	+0.123	−0.001	+1.937
Positive − Negative		+0.498 ***	+0.020 ***	+0.165 ***	+0.333 ***
All firm-years	142,071	+2.074	+0.125	+0.063	+2.011

Table 5 ■ Returns-earnings regressions, partitioned

The relation between returns and earnings, using firm-year observations over the period 1974 through 2009. Following Hayn (1995), we measure stock return (R_t) over the 12 months commencing with the fourth month of fiscal year t and earnings (X_t) as earnings before extraordinary items per share in year t. We scale X_t by price at the end of fiscal year $t-1$ (P_{t-1}). We regress R_t on X_t/P_{t-1} using a spline regression with a single knot located at $X_t/P_{t-1} = x^*$ [1-SPL]:

$$R_t = \alpha_{\mathrm{SPL}} + \beta_{\mathrm{L}}\left(\frac{X_t}{P_{t-1}}\right) + \beta_{\mathrm{IR}} \max\left(\frac{X_t}{P_{t-1}} - x^*,\ 0\right) + \varepsilon_t .$$

Additionally, we also estimate the analogous OLS regression equation [1-OLS]:

$$R_t = \alpha_{\mathrm{OLS}} + \beta_{\mathrm{OLS}}\left(\frac{X_t}{P_{t-1}}\right) + \varepsilon_t .$$

We compute one-tail contributions to convexity equal to the slope to the right of the spline knot $(r\_slope = \hat{\beta}_{\mathrm{R}} = \hat{\beta}_{\mathrm{L}} + \hat{\beta}_{\mathrm{IR}})$ and the negative of the slope to the left of the spline knot $(neg\_l\_slope = -\hat{\beta}_{\mathrm{L}})$. We compute two-tail measures of convexity equal to the difference in the right and left slopes $(rl\_slope\_diff = \hat{\beta}_{\mathrm{R}} - \hat{\beta}_{\mathrm{L}})$ and the vertical distance between the OLS fitted value and spline fitted value at the estimated spline knot ($ols\_spl\_dist = \hat{y} - y^*$). Panels A through D split the full sample into subsamples, partitioned on the four dimensions described in the optionality tree provided in Table 4, Panel A. The symbols ***, **, and * denote significance at the 1%, 5%, and 10% levels for two-tailed tests, respectively. We compute t-statistics using standard errors clustered by firm and year.

Panel A: Partitioned based on volatility

	Spline regression					OLS regression			
	β_{L} (n = 16,791)	β_{R} (n = 77,912)	$\beta_{\mathrm{R}} - \beta_{\mathrm{L}}$	x^*	y^*	α_{OLS} (n = 94,703)	β_{OLS} (n = 94,703)	$\hat{y} = \alpha + \beta x^*$	$\hat{y} - y^*$
High volatility (n = 47,357)	−0.146 ***	2.181 ***	2.327 ***	−0.020	0.023	0.197 ***	0.784 ***	0.181	0.159 ***
	−3.94	50.21	40.72			62.07	37.27		673.38
Low volatility (n = 47,346)	0.222 ***	1.693 ***	1.471 ***	−0.049	−0.056	0.056 ***	1.264 ***	−0.006	0.050 ***
	2.85	53.97	17.35			21.57	47.36		126.30
Test of differences (High − Low)	**−0.368** ***	**0.488** ***	**0.856** ***			0.141 ***	−0.480 ***		**0.109** ***
	−4.26	9.11	8.41			34.37	−14.11		63.09

	Two-tail measures of convexity		One-tail contributions to convexity	
	rl_slope_diff	*ols_spl_dist*	*neg_l_slope*	*r_slope*
High volatility	+2.327	+0.159	+0.146	+2.181
Low volatility	+1.471	+0.050	−0.222	+1.693
High − Low	**+0.856** ***	**+0.109** ***	**+0.368** ***	**+0.488** ***

Panel B: High volatility subsample, partitioned based on liquidation likelihood

	Spline regression					OLS regression			
	β_L	β_R	$\beta_R - \beta_L$	x^*	y^*	α_{OLS}	β_{OLS}	$\hat{y} = \alpha + \beta x^*$	$\hat{y} - y^*$
	(n = 14,111)	(n = 33,246)				(n = 47,357)	(n = 47,357)		
High liquidation likelihood (n = 23,842)	−0.092 **	1.971 ***	2.063 ***	−0.016	0.106	0.279 ***	0.697 ***	0.268	0.162 ***
	−2.07	33.73	28.08			59.64	27.51		499.06
Low liquidation likelihood (n = 23,515)	0.062	2.278 ***	2.216 ***	−0.040	−0.068	0.103 ***	1.157 ***	0.057	0.125 ***
	0.89	36.38	23.63			24.24	32.38		330.60
Test of differences (High − Low)	**−0.154** *	**−0.307** ***	**−0.153**			0.176 ***	−0.460 ***		**0.037** ***
	1.86	3.58	1.28			27.76	−10.50		23.02

	Two-tail measures of convexity		One-tail contributions to convexity	
	rl_slope_diff	*ols_spl_dist*	*neg_l_slope*	*r_slope*
High liquidation likelihood	+2.063	+0.162	+0.092	+1.971
Low liquidation likelihood	+2.216	+0.125	−0.062	+2.278
High − Low	**−0.153**	**+0.037** ***	**+0.154** *	**−0.307** ***

Panel C1: High volatility and low liquidation likelihood observations, partitioned based on investment real optionality

	Spline regression					OLS regression			
	β_L	β_R	$\beta_R - \beta_L$	x^*	y^*	α_{OLS}	β_{OLS}	$\hat{y} = \alpha + \beta x^*$	$\hat{y} - y^*$
	(n = 5,341)	(n = 18,174)				(n = 23,515)	(n = 23,515)		
High invest. real optionality (n = 7,872)	0.006	4.144 ***	4.138 ***	−0.015	−0.055	0.111 ***	1.192 ***	0.093	0.148 ***
	0.05	16.78	15.12			14.07	14.71		323.01
Low invest. real optionality (n = 15,643)	0.070	2.216 ***	2.146 ***	−0.040	−0.082	0.100 ***	1.157 ***	0.054	0.136 ***
	0.82	32.40	19.60			19.11	27.93		244.05
Test of differences (High − Low)	**−0.064**	**1.928** ***	**1.992** ***			0.011	0.035		**0.012** ***
	−0.44	7.52	6.76			1.12	0.38		17.81

	Two-tail measures of convexity		One-tail contributions to convexity	
	rl_slope_diff	*ols_spl_dist*	*neg_l_slope*	*r_slope*
High investment real optionality	+4.138	+0.148	−0.006	+4.144
Low investment real optionality	+2.146	+0.136	−0.070	+2.216
High − Low	**+1.992** ***	**+0.012** ***	**+0.064** ***	**+1.928** ***

Panel C2: High volatility and high liquidation likelihood observations, partitioned based on investment real optionality

	Spline regression					OLS regression			
	β_L (n = 9,416)	β_R (n = 14,426)	$\beta_R - \beta_L$	x^*	y^*	α_{OLS} (n = 23,842)	β_{OLS} (n = 23,842)	$\hat{y} = \alpha + \beta x^*$	$\hat{y} - y^*$
High invest. real optionality (n = 6,394)	−0.253 *** −2.62	2.818 *** 12.96	3.071 *** 12.91	−0.021	0.129	0.299 *** 27.59	0.538 *** 8.23	0.288	0.159 *** 262.59
Low invest. real optionality (n = 17,448)	−0.073 −1.44	2.006 *** 32.54	2.079 *** 26.10	−0.016	0.078	0.267 *** 50.66	0.755 *** 26.96	0.255	0.177 *** 468.85
Test of differences (High − Low)	**−0.180** * −1.65	**0.812** *** 3.59	**0.992** *** 3.95			0.032 *** 2.69	−0.217 *** −3.05		**−0.018** *** 25.13

	Two-tail measures of convexity		One-tail contributions to convexity	
	rl_slope_diff	*ols_spl_dist*	*neg_l_slope*	*r_slope*
High investment real optionality	+3.071	+0.159	+0.253	+2.818
Low investment real optionality	+2.079	+0.177	+0.073	+2.006
High − Low	**+0.992** ***	**−0.018** ***	**+0.180** *	**+0.812** ***

Panel D: High volatility, high liquidation likelihood, and high investment real optionality observations, partitioned based on market continuation outcome

	Spline regression					OLS regression			
	β_L (n = 3,676)	β_R (n = 2,718)	$\beta_R - \beta_L$	x^*	y^*	α_{OLS} (n = 6,394)	β_{OLS} (n = 6,394)	$\hat{y} = \alpha + \beta x^*$	$\hat{y} - y^*$
Positive continuation (n = 5,048)	−0.321 *** −2.88	3.021 *** 11.65	3.342 *** 11.84	−0.025	0.141	0.328 *** 25.20	0.537 *** 7.00	0.314	0.173 *** 226.60
Negative continuation (n = 1,346)	0.108 0.61	2.525 *** 6.80	2.417 *** 5.88	0.000	0.082	0.199 *** 10.52	0.701 *** 5.88	0.199	0.117 *** 117.51
Test of differences (High − Low)	**−0.429** ** −2.06	**0.496** 1.09	**0.925** * 1.86			0.129 *** 5.61	−0.164 −1.16		**0.056** *** 43.01

	Two-tail measures of convexity		One-tail contributions to convexity	
	rl_slope_diff	*ols_spl_dist*	*neg_l_slope*	*r_slope*
Positive continuation	+3.342	+0.173	+0.321	+3.021
Negative continuation	+2.417	+0.117	−0.108	+2.525
Positive − Negative	**+0.925** *	**+0.056** ***	**+0.429** **	**+0.496**

Table 6 ■ Measures of convexity

Panel A displays the four convexity measures for each path in the optionality tree. Panel B summarizes the incremental R^2 contribution to the returns-earnings regression from each of our spline partition and optionality tree partitions. The incremental R^2 is based on the incremental regression sum of squares provided by the each of the spline partition and optionality tree partitions in explaining the total sum of squares of the basic OLS regression for the full sample.

Panel A: The four convexity measures by path [*rl_slope_diff* | *ols_spl_dist* | *neg_l_slope* | *r_slope*]

<table>
<tr><th>VOLATILITY</th><th>LIQUIDATION LIKELIHOOD</th><th>INVESTMENT REAL OPTIONALITY</th><th>MARKET CONTINUATION</th><th rowspan="2">PATH</th></tr>
<tr><th>stdret</th><th>inverse price</th><th>high_rd</th><th>equity_issue</th></tr>
<tr><td rowspan="8">High
[2.327 | 0.159 | 0.146 | 2.181]</td><td rowspan="4">Low
[2.216 | 0.125 | −0.062 | 2.278]</td><td rowspan="2">High
[4.138 | 0.148 | −0.006 | 4.144]</td><td>Yes [4.331 | 0.153 | −0.014 | 4.345]</td><td>(1)</td></tr>
<tr><td>No [3.539 | 0.138 | 0.008 | 3.531]</td><td>(2)</td></tr>
<tr><td rowspan="2">Low
[2.146 | 0.136 | −0.070 | 2.216]</td><td>Yes [2.577 | 0.161 | 0.038 | 2.539]</td><td>(3)</td></tr>
<tr><td>No [1.900 | 0.125 | −0.142 | 2.042]</td><td>(4)</td></tr>
<tr><td rowspan="4">High
[2.063 | 0.162 | 0.092 | 1.971]</td><td rowspan="2">High
[3.071 | 0.159 | 0.253 | 2.818]</td><td>Yes [3.342 | 0.173 | 0.321 | 3.021]</td><td>(5)</td></tr>
<tr><td>No [2.417 | 0.117 | −0.108 | 2.525]</td><td>(6)</td></tr>
<tr><td rowspan="2">Low
[2.079 | 0.177 | 0.073 | 2.006]</td><td>Yes [2.526 | 0.206 | 0.186 | 2.340]</td><td>(7)</td></tr>
<tr><td>No [1.921 | 0.169 | 0.020 | 1.901]</td><td>(8)</td></tr>
<tr><td rowspan="8">Medium
[1.840 | 0.102 | −0.167 | 2.006]</td><td rowspan="4">Low
[1.639 | 0.049 | −0.206 | 1.845]</td><td rowspan="2">High
[3.077 | 0.115 | −0.008 | 3.085]</td><td>Yes [3.538 | 0.145 | 0.222 | 3.316]</td><td>(9)</td></tr>
<tr><td>No [2.701 | 0.085 | −0.292 | 2.993]</td><td>(10)</td></tr>
<tr><td rowspan="2">Low
[1.832 | 0.075 | −0.010 | 1.841]</td><td>Yes [2.250 | 0.139 | 0.384 | 1.866]</td><td>(11)</td></tr>
<tr><td>No [1.757 | 0.065 | −0.208 | 1.965]</td><td>(12)</td></tr>
<tr><td rowspan="4">High
[1.872 | 0.119 | −0.223 | 2.095]</td><td rowspan="2">High
[4.023 | 0.163 | 0.035 | 3.988]</td><td>Yes [4.383 | 0.201 | 0.399 | 3.984]</td><td>(13)</td></tr>
<tr><td>No [3.457 | 0.162 | −0.142 | 3.599]</td><td>(14)</td></tr>
<tr><td rowspan="2">Low
[1.851 | 0.121 | −0.259 | 2.110]</td><td>Yes [2.114 | 0.128 | −0.317 | 2.431]</td><td>(15)</td></tr>
<tr><td>No [1.809 | 0.124 | −0.206 | 2.015]</td><td>(16)</td></tr>
<tr><td rowspan="8">Low
[1.471 | 0.050 | −0.222 | 1.693]</td><td rowspan="4">Low
[0.937 | 0.001 | −0.563 | 1.501]</td><td rowspan="2">High
[1.991 | 0.036 | −0.169 | 2.160]</td><td>Yes [1.860 | 0.038 | −0.290 | 2.150]</td><td>(17)</td></tr>
<tr><td>No [2.784 | 0.060 | 0.295 | 2.489]</td><td>(18)</td></tr>
<tr><td rowspan="2">Low
[0.984 | 0.001 | −0.533 | 1.518]</td><td>Yes [1.083 | 0.007 | −0.474 | 1.557]</td><td>(19)</td></tr>
<tr><td>No [0.947 | −0.002 | −0.574 | 1.521]</td><td>(20)</td></tr>
<tr><td rowspan="4">High
[1.641 | 0.075 | −0.161 | 1.802]</td><td rowspan="2">High
[3.022 | 0.114 | −0.423 | 3.445]</td><td>Yes [3.160 | 0.111 | −0.768 | 3.928]</td><td>(21)</td></tr>
<tr><td>No [3.203 | 0.123 | −0.243 | 3.446]</td><td>(22)</td></tr>
<tr><td rowspan="2">Low
[1.692 | 0.080 | −0.102 | 1.794]</td><td>Yes [1.283 | 0.047 | −0.541 | 1.824]</td><td>(23)</td></tr>
<tr><td>No [1.860 | 0.103 | 0.044 | 1.816]</td><td>(24)</td></tr>
</table>

Table 6 ■ Measures of convexity (continued)

Panel B: Incremental R^2 contribution by spline partition and optionality tree partitions

	Full sample	Volatility	Liquidation likelihood	Investment real optionality	Market continuation
OLS R^2	0.0513	0.0618	0.0730	0.0741	0.0776
Spline R^2	0.0869	0.0942	0.0998	0.1049	0.1112

Table 7 ■ Intertemporal and cross-sectional variation in convexity measures

We compute convexity values for certain subsamples within the set of high volatility tercile firms. In Panel A, we compare the subsample of high volatility firms in the technology bubble period (defined as 1995–1999) with non-bubble period high volatility firms. In Panel B, we classify firms into one of five lifecycle categories, using a modification of the Dickinson (2011) approach. The symbols ***, **, and * denote significance at the 1%, 5%, and 10% levels for two-tailed tests, respectively.

Panel A: Non-bubble years vs. bubble years

	Spline knot		Two-tail measures of convexity		One-tail contributions to convexity	
	*x**	*y**	*rl_slope_diff*	*ols_spl_dist*	*neg_l_slope*	*r_slope*
Non-bubble years	−0.021	−0.007	+2.335	+0.164	+0.122	+2.213
Bubble years	−0.022	+0.105	+2.728	+0.166	+0.326	+2.402
			+0.393 ***	+0.002 ***	+0.204 *	+0.189

Panel B: Modified Dickinson (2011) life cycle analysis

		Spline knot		Two-tail measures of convexity		One-tail contributions to convexity	
	n ♦	*x**	*y**	*rl_slope_diff*	*ols_spl_dist*	*neg_l_slope*	*r_slope*
[1] Introduction	4,828	−0.019	−0.025	+3.664	+0.180	+0.460	+3.204
[2] Growth	12,557	−0.010	+0.012	+3.595	+0.165	+0.289	+3.306
[3] Mature	10,027	−0.006	+0.068	+3.217	+0.193	+0.309	+2.908
[4] Shakeout	2,534	−0.012	+0.145	+2.186	+0.145	+0.150	+2.036
[5] Decline	1,549	−0.030	−0.010	+2.952	+0.222	+0.787	+2.165
	31,495						
[1] + [2] + [3]	27,412	−0.014	0.014	+3.465	+0.189	+0.350	+3.115
[4] + [5]	4,083	−0.020	0.086	+2.531	+0.178	+0.400	+2.131
	31,495			+0.934 ***	+0.011	−0.050	+0.984 ***

♦ The total sample of 31,495 high volatility observations is less than the high volatility tercile sample of $n = 47{,}357$ because we require cash flow statement data, which are available only after the passage of FAS 95 (ASC 230) in 1987.

Table 8 ■ Financing prediction models

The relation between convexity and [1] the equity market's purchase of the firm's continuation option (Panel A), and [2] the debt market's purchase of the firm's continuation option (Panel B). We estimate various specifications of the following logistic regression models:

$$\begin{aligned}\Pr\left(\text{equity\_issue}_{t+1}=1\right) &= \alpha+\beta_1\left(\text{stdret}_t\right)+\beta_2\left(\text{loss\_seq}_t\right)+\beta_3\left(\text{loss\_seq}_t\times\text{stdret}_t\right)\\ &+\beta_4\left(\text{leverage}_t\right)+\beta_5\left(\text{fdeficit}_t\right)+\beta_6\left(\text{log\_mve}_t\right)+\beta_7\left(\text{ols\_spl\_dist}\right)+\beta_8\left(\text{loss\_seq}_t\times\text{ols\_spl\_dist}\right)\\ &+\beta_9\left(\text{rl\_slope\_diff}\right)+\beta_{10}\left(\text{loss\_seq}_t\times\text{rl\_slope\_diff}\right)+\beta_{11}\left(\text{neg\_l\_slope}\right)+\beta_{12}\left(\text{loss\_seq}_t\times\text{neg\_l\_slope}\right)\\ &+\beta_{13}\left(\text{r\_slope}\right)+\beta_{14}\left(\text{loss\_seq}_t\times\text{r\_slope}\right)+\beta_{15}\left(\text{equity\_issue}_t\right)+\varepsilon.\end{aligned} \quad [2]$$

$$\begin{aligned}\Pr\left(\text{debt\_issue}_{t+1}=1\right) &= \alpha+\gamma_1\left(\text{stdret}_t\right)+\gamma_2\left(\text{loss\_seq}_t\right)+\gamma_3\left(\text{loss\_seq}_t\times\text{stdret}_t\right)\\ &+\gamma_4\left(\text{leverage}_t\right)+\gamma_5\left(\text{fdeficit}_t\right)+\gamma_6\left(\text{log\_mve}_t\right)+\gamma_7\left(\text{ols\_spl\_dist}\right)+\gamma_8\left(\text{loss\_seq}_t\times\text{ols\_spl\_dist}\right)\\ &+\gamma_9\left(\text{rl\_slope\_diff}\right)+\gamma_{10}\left(\text{loss\_seq}_t\times\text{rl\_slope\_diff}\right)+\gamma_{11}\left(\text{neg\_l\_slope}\right)+\gamma_{12}\left(\text{loss\_seq}_t\times\text{neg\_l\_slope}\right).\\ &+\gamma_{13}\left(\text{r\_slope}\right)+\gamma_{14}\left(\text{loss\_seq}_t\times\text{r\_slope}\right)+\gamma_{15}\left(\text{debt\_issue}_t\right)+\varepsilon.\end{aligned} \quad [3]$$

***, **, and * denote significance at the 1%, 5%, and 10% levels for two-tailed tests, respectively. Please refer to Appendix A for variable definitions. We compute significance using standard errors clustered at the firm-year level. The sample size of n = 138,399 is smaller than the full sample size of 142,071 because we are predicting equity and debt issuance for the leading year, and thus lose the most recent year (2009) of observations in our sample.

Table 8 ■ Financing prediction models (continued)

Panel A: Predicting equity issuance

		Response variable = equity_issue$_{t+1}$ (n = 138,399)				
		[1]	[2]	[3]	[4]	[5] ♣
Intercept		−0.966 ***	−2.032 ***	−2.504 ***	−2.939 ***	−1.910 ***
		−37.76	−66.46	−75.53	−68.11	−43.71
stdret$_t$	♦	18.828 ***	−7.915 ***	−6.374 ***	−5.622 ***	9.518 ***
		36.64	−14.09	−11.96	−10.48	12.61
loss_seq$_t$	−	0.116 ***	−0.697 ***	−0.893 ***	−0.575 ***	−0.166 ***
		7.00	−17.81	−19.88	−10.77	−4.97
loss_seq$_t$ × stdret$_t$	♦	−0.450	0.852 **	1.060 ***	0.196	−0.364
		−1.14	2.68	3.00	0.61	−0.70
leverage$_t$	?	−0.099 ***	−0.082 ***	0.079 ***	0.080 ***	0.037 ***
		−13.39	−11.02	10.92	11.12	4.14
fdeficit$_t$	+	7.146 ***	7.114 ***	6.980 ***	7.014 ***	6.585 ***
		78.84	76.26	73.29	73.26	62.02
log_mve$_t$	?	0.079 ***	0.190 ***	0.104 ***	0.101 ***	0.034 ***
		26.87	55.09	32.73	31.66	9.43
ols_spl_dist	+		11.739 ***			
			75.29			
loss_seq$_t$ × ols_spl_dist	+		4.806 ***			
			18.12			
rl_slope_diff	+			1.022 ***		
				84.15		
loss_seq$_t$ × rl_slope_diff	+			0.365 ***		
				16.98		
neg_l_slope	+				0.616 ***	0.204 ***
					24.59	4.47
loss_seq$_t$ × neg_l_slope	+				0.714 ***	0.115 **
					18.37	2.53
r_slope	+				1.203 ***	0.183 ***
					67.10	12.16
loss_seq$_t$ × r_slope	+				0.244 ***	0.078 ***
					9.87	7.14
equity_issue$_t$	+					2.094 ***
						148.41
ROC area		0.711	0.749	0.773	0.779	0.824

Table 8 ■ Financing prediction models (continued)

Panel B: Predicting debt issuance

		Response variable = debt_issue$_{t+1}$ (n = 138,399)				
		[1]	[2]	[3]	[4]	[5] ♣
Intercept		−0.760 ***	−0.582 ***	−0.279 ***	−0.044	−0.397 ***
		−28.51	−20.85	−9.36	−1.18	−8.93
stdret$_t$	−	−21.430 ***	−15.635 ***	−11.525 ***	−12.191 ***	−14.496 ***
		−36.94	−21.52	−18.04	−19.03	−19.58
loss_seq$_t$	−	−0.107 ***	0.147 ***	0.554 ***	0.464 ***	0.371 ***
		−3.79	4.76	16.92	11.48	7.10
loss_seq$_t$ × stdret$_t$	−	−5.880 ***	−5.027 ***	−3.561 ***	−3.159 ***	−3.118 ***
		−7.83	−5.91	−5.27	−4.56	−3.54
leverage$_t$	?	0.395 ***	0.389 ***	0.306 ***	0.304 ***	0.228 ***
		52.28	51.62	42.19	41.84	27.62
fdeficit$_t$	+	10.513 ***	10.629 ***	11.620 ***	11.635 ***	13.135 ***
		70.61	71.42	73.41	73.58	66.60
log_mve$_t$	?	0.099 ***	0.078 ***	0.094 ***	0.097 ***	0.088 ***
		32.68	23.98	30.93	31.80	28.72
ols_spl_dist	−		−2.200 ***			
			−14.40			
loss_seq$_t$ × ols_spl_dist	−		−1.888 ***			
			−7.20			
rl_slope_diff	−			−0.358 ***		
				−32.56		
loss_seq$_t$ × rl_slope_diff	−			−0.277 ***		
				−18.43		
neg_l_slope	−				−0.091 ***	0.116 **
					−3.80	2.77
loss_seq$_t$ × neg_l_slope	−				−0.415 ***	−0.695 ***
					−9.16	−8.82
r_slope	−				−0.450 ***	−0.378 ***
					−30.91	−22.03
loss_seq$_t$ × r_slope	−				−0.247 ***	−0.243 ***
					−14.94	−11.52
debt_issue$_t$	+					0.789 ***
						55.78
ROC area		0.823	0.823	0.830	0.831	0.844

♦ In the absence (presence) of convexity, we predict a positive (negative) coefficient for stdret and its interaction with loss_seq.

♣ Specifications [1] through [4] use the convexity measures associated with all four partitions (i.e., 24 paths) displayed in the optionality tree in Table 4; specification [5] uses convexity measures associated with the first three partitions (i.e., 12 paths) and includes equity_issue$_t$ (debt_issue$_t$) when predicting equity_issue$_{t+1}$ (debt_issue$_{t+1}$).