Not Only What But also When: A Theory of Dynamic Voluntary Disclosure*

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Abstract

We examine a dynamic model of voluntary disclosure of multiple pieces of private information. In our model, a manager of a firm who may learn multiple signals over time interacts with a competitive capital market and maximizes payoffs that increase in both period prices. We show (perhaps surprisingly) that in equilibrium later disclosures are interpreted more favorably even though the time the manager obtains the signals is independent of the value of the firm. We also provide sufficient conditions for the equilibrium to be in threshold strategies.

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1 Introduction

We study a dynamic model of voluntary disclosure of information by a potentially informed agent. The extant theoretical literature on voluntary disclosure focuses on static models in which an interested party (e.g., a manager of a firm) may privately observe a single piece of private information (e.g., Grossman 1981, Milgrom 1981, Dye 1985, and Jung and Kwon 1988) or dynamic models in which the disclosure timing does not play a role (e.g., Shin 2003, 2006) as the manager's decision is what to disclose but not when to disclose it. Corporate disclosure environments, however, are characterized by multi-period and multi-dimensional flows of information from the firm to the market, where the information asymmetry between the firm and the capital market can be with respect to whether, when, and what relevant information the firm might have learned. For example, firms with ongoing R&D projects can obtain new information about the state of their projects, where the time of information arrival and its content is unobservable to the market. This is common, for example, in pharmaceutical companies that get results of a drug's clinical trial (prior to FDA approval). Such results are not required to be publicly disclosed in a timely manner and investors' beliefs about the result of a drug's clinical trial may have a great effect on the firm's price. The multidimensional nature of the disclosure game (multi-period and multi-signal) plays a critical role in shaping the equilibrium; e.g., when deciding whether to disclose one piece of information the agent must also consider the possibility of learning and potentially disclosing a new piece of information in the future.

In order to study a dynamic model of voluntary disclosure, we extend Dye's (1985) and Jung and Kwon's (1988) voluntary disclosure model with uncertainty about information endowment to a two-period, two-signal setting. We describe the potentially informed agent as a manager of a publicly traded firm. In our model, the manager cares about stock prices in both periods and he may receive up to two private signals about the value of the firm. In each period, the manager may voluntarily disclose any subset of the signals he has received but not yet disclosed. Our model demonstrates how dynamic considerations shape the disclosure strategy of a privately informed agent and the market reactions to what he releases and when. Absent information asymmetry, the firm's price at the end of the second period is independent of the arrival and disclosure times of the firm's private information. Nevertheless, we show that in equilibrium, the market price depends not only on what information has been disclosed so far, but also on when it was disclosed. In particular, we show that the price at the end of the second period given disclosure of one signal is *higher if the signal is disclosed later* in the game. This result might be counter intuitive, as one might expect the market to reward the manager for early disclosure of information, since then he seems less likely to be "hiding something."

The intuition behind this result can be explained through a variant of the original static voluntary disclosure model of Dye (1985). In Dye (1985) the agent either learns nothing or learns a single signal with some probability. The equilibrium in that model is a threshold strategy where the price upon non-disclosure matches the price from disclosing the threshold type. Consider a variant of this game in which the disclosure is done in two steps. In the first step the agent follows an arbitrary exogenously determined disclosure policy. As a result the set of informed agents who have not disclosed the signal is some set B. In the second step the agent optimizes so that he reports if and only if doing so improves his payoff. Again, the equilibrium prices upon upon non-disclosure matches the price if the threshold type is disclosed. But these prices are affected by disclosure in the first step, i.e. by the set B. We shall see that the smaller is B (in set-inclusion sense), the *higher* the equilibrium non-disclosure price (see Lemma 2 for a proof). For example, consider two possible cases for the first step where $B' \supset B''$, that is, in case of B'' the agent follows a more aggressive disclosure in the first step. Our result is that even if all signals in $B' \backslash B''$ are higher than all signals in B'', the non-disclosure price in case of B' is lower. Moreover, if some signals in $B' \backslash B''$ are smaller than the second-step threshold in case of B', the ranking of prices given non-disclosure is strict. In words, the more aggressive disclosure policy results in a higher inference for those who did not disclose in the second step.

How is this related to our result? Consider the following two histories (on the equilibrium path) in which the manager discloses only one signal, x. In history 1, the manager disclosed x at $t = 1$ while in history 2 he disclosed x at $t = 2$. The equilibrium price at $t = 2$ depends on the market belief about the value of the other signal that the manager may have received, signal, y. In period 2, given that x is revealed, the agent reveals y if it increases current price relative to the price given no disclosure of y, as in the second step in our hypothetical game. The first step in the hypothetical game corresponds to the option to reveal y at different private histories. For example, to reveal y at $t = 1$. Our proof relies on comparing the aggressiveness of the disclosure policy for y under different scenarios about when the agent learned y given the observed history when he revealed x. A private history that plays a key role in our proof is the following: if the agent reveals x at $t = 1$, investors know that he could not have known only y at $t = 1$. However, if he reveals x at $t = 2$, they cannot rule out that at $t = 1$ he knew only y. Moreover, investors know that if the agent knows only one of the signals at $t = 1$, he discloses it if it is high enough. So when x is disclosed at $t = 2$, it implies that some possible realizations of y can be excluded. Even though the excluded values are relatively high realizations, it still leads to a positive inference, especially if the revealed x is high.

In Section 3, we formalize and extend this intuition to establish the main result of the paper. We argue that later disclosure receives a better interpretation provided that the equilibrium is monotone and symmetric. To further characterize the strategic behavior and market inferences in our model, in Section 4 we discuss the main strategic considerations in equilibrium and establish existence of threshold equilibria under suitable conditions.[1].

1.1 Related Literature

The voluntary disclosure literature goes back to Grossman and Hart (1980), Grossman (1981), and Milgrom (1981), who established the "unraveling result," which states that under certain assumptions (including: common knowledge that the agent is privately informed, disclosing is costless, and information is verifiable) all types disclose their information in equilibrium. In light of companies' propensity to withhold some private information, the literature on voluntary disclosure evolved around settings in which the unraveling result does not prevail. The two major streams of this literature are: (*i*) assuming that disclosure is costly (pioneered by Jovanovic 1982 and Verrecchia 1983) and (*ii*) investors' uncertainty about information endowment (pioneered by Dye 1985 and Jung and Kwon 1988). Our model follows Dye (1985) and Jung and Kwon (1988) and extends it to a multi-signal and a multi-period setting.

As mentioned in the introduction, in spite of the vast literature on voluntary disclosure, very little has been done on multi-period settings and on multi-signal settings.[2]

To the best of our knowledge the only papers that study multi-period voluntary disclosure are Shin (2003, 2006), Einhorn and Ziv (2008), and Beyer and Dye (2011). The settings studied in these papers as well as the dynamic considerations of the agents are very different from ours. Shin

[1] In most of the existing voluntary disclosure literature (e.g., Verrecchia 1983, Dye 1985, Acharya et al. 2011), the equilibrium always exists, is unique, and is characterized by a threshold strategy. In our model, due to multiple periods and signals, existence of a threshold equilibrium is not guaranteed, and therefore we provide sufficient conditions for existence (similar to Pae 2005).

[2] For example, this gap in the literature is pointed out in a survey by Hirst, Koonce, and Venkataraman (2008), who write "much of the prior research ignores the iterative nature of management earnings forecasts."

(2003, 2006) studies a setting in which a firm may learn a binary signal for each of its independent projects, where each project may either fail or succeed. In this binary setting, Shin (2003, 2006) studies the "sanitization" strategy, under which the agent discloses only the good (success) news. The timing of disclosure does not play a role in such a setup. Einhorn and Ziv (2008) study a setting in which in each period the manager may obtain a single signal about the period's cash flows, where at the end of each period the realized cash flows are publicly revealed. If the agent chooses to disclose his private signal, he incurs some disclosure costs. Acharya, DeMarzo, and Kremer (2011) examine a dynamic model in which a manager learns one piece of information at some random time and his decision to disclose it is affected by the release of some external news. They show that a more negative external signal is more likely to trigger the release of information by the firm. Perhaps surprisingly this clustering effect is present only in a dynamic model and not in a static one. Given that the firm may learn only one piece of information the effect that we study in our paper cannot be examined in their model. Finally, Beyer and Dye (2011) study a reputation model in which the manager may learn a single private signal in each of the two periods. The manager can be either "forthcoming" and disclose any information he learns or he may be "strategic." At the end of each period, the firm's signal/cash flow for the period becomes public and the market updates beliefs about the value of the firm and the type of the agent. Importantly, the option to "wait for a better signal" that is behind our main result is not present in any of these papers.

Our paper also adds to the understanding of management's decision to selectively disclose information. Most voluntary disclosure models assume a single signal setting, in which the manager can either disclose all of his information or not disclose at all. In practice, managers sometimes voluntarily disclose part of their private information while concealing another part of their private information. To the best of our knowledge, the only exceptions in the voluntary disclosure literature in which agents may learn multiple signals are Shin (2003, 2006), which we discussed above, and Pae (2005). The latter considers a single-period setting in which the agent can learn up to two signals. We add to Pae (2005) dynamic considerations, which are again crucial for creating the option value of waiting for a better signal.

Bhattacharya and Ritter (1983) examine another aspect of disclosure to capital markets. In their model multiple firms compete in an R&D race. A firm may have information about better technology that enables it to advance faster. The trade-off in that paper is that revealing information about this technology leads to better financing terms but at the same time reduces the firm's technological advantage over competing firms.

2 The Model

Consider the following dynamic voluntary disclosure game. There is an agent, who we refer to as a manager of a publicly traded company, and a competitive market of risk neutral investors. The value of the company is a realization of a random variable, V, and V is not known to the market or the manager. All agents share a common prior over the distribution of V. There are two signals of V, which we denote by X and Y. Conditional on V these signals are identically and independently distributed over $\mathbb{R}^2$ according to some atomless distribution. We assume that the support of the conditional distribution of a signal is independent of the realization of V and that the density of that distribution is positive on this support.

We denote the expected value of V given the realizations of the two signals, (x, y), by:

$$E[V|X = x, Y = y] = P(x, y) = P(y, x).$$

We assume that P is continuous and strictly increasing in both arguments.

The game has two periods, $t \in \{1, 2\}$. At the beginning of period 1 the manager *privately* learns each of the signals with probability p. Learning a signal is independent across the two signals, so that the probability of learning both signals at $t = 1$ is p^2. Learning a signal is also independent of the value of any of the signals or the value of the company. In the beginning of period 2 the manager learns with probability p any signal that he has not yet learned in period 1.[3]

Each period, after potentially learning some signals, the manager decides whether to reveal some or all of the signals he has learned and not yet disclosed: disclosure is voluntary and can be selective. We follow Grossman (1981), Milgrom (1981) and Dye (1985) and assume that: (*i*) the agent cannot credibly convey the fact that he did not obtain a signal, and (*ii*) any disclosure is truthful (or verifiable at no cost) and does not impose a direct cost on the manager or the firm.

A public history at time t contains the set of signals that the agent has revealed and the time each signal was revealed, (t_x, t_y). We denote the public history by h_t^P and let $H_t^P = \{\emptyset, (x, t_x), (y, t_y), (x, y, t_x, t_y)\}$ denote the set of potential public histories, where $\emptyset$ denote a history in which no disclosure has been made. The market does not know when an agent has learned a

[3] All the model's analysis and results are robust to the introduction of a third period in which the private signals learned by the manager are publicly revealed.

signal. For example, if the agent reveals a signal x in period 2, the market cannot directly observe whether the manager learned that signal in period 1 or 2.

Investors observe only the public history. The agent observes both the public and a private history. Agent's private history at the beginning of period 1 is the signals he has learned so far, $h_1^A \in H_1^A = \{\emptyset, x, y, (x,y)\}$. At the beginning of period 2 a private history is the signals that the agent has learned and when he has learned them, $h_2^A \in H_2^A = \{\emptyset, (x,\tau_x), (y,\tau_y), (x,y,\tau_x,\tau_y)\}$, where (τ_x, τ_y) denote the times the agent has learned the signals X and Y, respectively. We denote by $\tau_x, \tau_y > 2$ the case that the agent did not learn the corresponding signal.

A (behavioral) strategy of the agent is a disclosure policy which is a mapping from histories (public and private since the agent observes both) into a decision whether to reveal any of the signals he has observed so far and not disclosed yet.

We model investors in a reduced form: given the public history, they form beliefs about the value of the firm and set the market price at time t equal to:

$$P_t\left(H_t^P\right) \equiv E\left[V|H_t^P\right] = E\left[P\left(x,y\right)|H_t^P\right].$$

Note that conditional on the agent revealing both signals, the market price is $P_t\left(x,y,t_x,t_y\right) = P\left(x,y\right)$ and it is independent of when the signals were disclosed. This follows from the fact that upon revealing both signals there is no information asymmetry about V.[4] However, in other cases investors form beliefs based on the equilibrium strategy of the agent and will infer that the agent might have learned some signals and decided not to reveal them. For example, when only one signal, e.g., x, has been revealed the price will be

$$P_t\left(x,t_x\right) = E_y\left[P\left(x,y\right)|H_t^P = \left(x,t_x\right)\right],$$

where the beliefs over the second signal, y, are formed consistently with Bayes rule and the equilibrium strategy of the agent, whenever possible.

We assume the manager maximizes a payoff function:

$$U(P_1\left(H_1^P\right), P_2\left(H_2^P\right)),$$

that is continuous and strictly increasing in both prices. The interpretation is that the manager's compensation is increasing in each period's stock price (and/or that the probability of losing the job

[4] Recall the assumption that τ_x and τ_y are independed of V.

is decreasing in each period's stock price and the manager strictly dislikes being fired). In Section 4 we analyze the model with additional distributional assumptions and with $U(P_1\left(H_1^P\right), P_2\left(H_2^P\right)) = P_1\left(H_1^P\right) + P_2\left(H_2^P\right)$ to provide additional results.

A (perfect Bayesian) equilibrium is a profile of disclosure policies of the agent and a set of price functions $\{P_t(\emptyset), P_t(x, t_x), P_t(y, t_y), P(x, y)\}$ (both on and off the equilibrium path) such that the agent optimizes given the price functions and the prices are consistent with the strategy of the agent by applying Bayes rule whenever possible. The equilibrium is *monotone* if the price function $P_t(x, t_x)$ is increasing in x for all t and t_x. We restrict our analysis to *symmetric monotone* equilibria *in pure strategies* that is, monotone equilibria in which $P_t(x, t_x) = P_t(y, t_y)$ (i.e. the price does not depend on which signal has been revealed) and the agent's disclosure policy is deterministic (i.e. on the equilibrium path the agent does not randomize whether to reveal a signal or not given the history).

Remark 1 *We assume that investors can tell which signal (X or Y) is disclosed. This applies to many real world applications where signals correspond to different dimensions of the firm's business. For example, signals may correspond to information about revenues and costs, represent information about two different markets or correspond to two different projects of the firm. That said, given the symmetric setup and our focus on symmetric equilibria, the equilibrium outcomes we describe coincide with equilibrium outcomes in a game where investors cannot tell which signal is disclosed. For example, X and Y may be two signals about future sales of the company and the agent may obtain them over time.*

Figure 1 summarizes the sequence of events in the model.

3 Later Disclosures Receive Better Responses

In this section we present our main result: if we compare two public histories in which only one signal is revealed but at different times, the market price is higher in the history with later disclosure.[5] In other words, the market forms its beliefs based on what is revealed *and also when it is revealed* despite the value V being independent of the times the agent learns the signals.

[5] Since we have only 2 signals, to show the effect of time of disclosure on equilibrium prices, we have to focus on the histories with one signal revealed.

The manager learns ach of the signals, X and Y, with probability *p* and decides what subsets of the signals he learned to disclose. At the end of the period investors set the stock price equal to their expectation of the firm's value, $P_1(H_1^P)$.

Each signal that has not yet been learned at *t=1* is obtained by the manager with probability *p*. The manager may disclose a subset of the signals he has received but not yet disclosed at *t=1*. At the end of the period investors set the stock price equal to their expectation of the firm's value, $P_2(H_2^P)$.

Figure 1: Timeline

As mentioned above, we focus on symmetric monotone equilibria in pure strategies. Without loss of generality, we focus on histories such that either X is disclosed before Y or both signals are not disclosed, that is, $t_x \leq t_y$.

Theorem 1 *Consider any symmetric monotone PBE in pure strategies in which public histories* $h_2^P = (x, 1)$ *and* $\hat{h}_2^P = (x, 2)$ *are on the equilibrium path. Then:*

$$P_2(x, 2) \geq P_2(x, 1),$$

i.e., in period 2 *the price upon revelation of only one signal is higher if that signal was revealed later.*

Theorem 1 characterizes a property of any symmetric monotone PBE in pure strategies. In the rest of this section we refer to this class of PBE as "equilibrium." In Section 4 we demonstrate the existence of a threshold equilibrium that has all these assumed properties. Moreover, we show in Section 4 that the effect of later disclosure on the price at $t = 2$ is strict for a range of signals; that there exists an x' such that $P_2(x, 2) > P_2(x, 1)$ for all $x > x'$ (and $(x, 1), (x, 2)$ are public histories on the equilibrium path).

We prove Theorem 1 via a series of lemmas. Some of the proofs are in the Appendix, but we try to present the main intuition in the remainder of this section.

We start by noting that at $t = 2$, since this is the last period, an agent that revealed one signal is myopic and reveals the second signal if and only if it improves the agent's payoff at $t = 2$ relative to non-disclosure of the second signal. That is:

Lemma 1 *In any equilibrium, conditional on revealing x (at any time), the manager reveals y at $t = 2$ if and only if $P(x, y) \geq P_2(x, t_x)$.*[6]

Given that $P(x, y)$ is increasing in y, the above lemma implies that at $t = 2$ the agent follows a threshold strategy by disclosing $y > Y_{x,t_x}$, where Y_{x,t_x} is defined by $P_2(x, t_x) = P(x, Y_{x,t_x})$.[7]

A key concept that we define and use is that of *potential disclosers*. The set of potential disclosers is defined as the set of types who, in equilibrium, learned Y either at $t = 1$ or $t = 2$, disclosed $X = x$ at either $t = 1$ or $t = 2$ and did not disclose Y at $t = 1$, i.e. types such that $\{t_x \leq 2,\ \tau_y \leq 2 \text{ and } t_y > 1\}$. This is the set of agents whose behavior is described by Lemma 1. The set of potential disclosers can be obtained by starting with the set of *informed (who learned* Y) and eliminating types that, on the equilibrium path, would have: (i) disclosed y at $t = 1$, (ii) disclosed y but not x, and (iii) preferred to disclose nothing given x and y. In subsections 3.2 and 3.3 we characterize the set of potential disclosers for h_2^P and $\hat{h}_2^P$, respectively.

Our proof of Theorem 1 follows from the comparison of the sets of potential disclosers for the two histories. Why are the sets of potential disclosers important for comparing $P_2(x, t_1)$ and $P_2(x, t_2)$? Prices at $t = 2$ are determined as follows. For any of the histories, start with two possibilities: either the agent does not know y, i.e. he is *uninformed* (which happens with an ex-ante probability $(1-p)^2$), or he is *informed* (and learned y either at time 1 or 2). Then, in case he is informed, exclude all realizations of Y that are inconsistent with equilibrium behavior given the history. This can be done in two steps: first exclude all types other than the potential disclosers and then apply Lemma 1 to remove additional types. That leaves only types (τ_y, y) that are consistent with the history and equilibrium strategies and we can compute the price as the expected value of $P(x, y)$ over these types (given the disclosed value of X and the conditional distribution of Y given X). We describe this procedure in greater detail in subsection 3.4. A difficulty in computing the equilibrium $P_2(x, t_x)$ is that it is a solution to a fixed-point problem: the price depends on the disclosure policy and vice versa (i.e., Y_{x,t_x} and $P_2(x, t_x)$ are interdependent). This is why it is useful to divide the exclusion of types after the two histories into the identification of the potential disclosers and the application of Lemma 1, where the second step captures the fixed-point reasoning at $t = 2$.

[6] To simplify the exposition, throughout this section we assume that an agent who is indifferent will disclose his information, but it is without loss of generality.

[7] Existence and uniqueness of Y_{x,t_x} follows from a) $P(x, y)$ is increasing and continuous in y; b) $P_t(x, t_x)$ is the expected value of $P(x, y)$ conditional on the equilibrium beliefs about y so it is in the range of $P(x, .)$.

3.1 Generalized Minimum Principle

In this subsection we introduce Lemma 2, which is an extension of the minimum principle in Acharya, DeMarzo, and Kremer (2011), and which will help us characterize the equilibrium prices.[8]

Given sets A and B, and an increasing continuous function g, define $S_{A,B}$ as

$$S_{A,B} \equiv A \cup \{B \cap \{y : g(y) < E[g(y)|y \in S_{A,B}]\}\}.$$

Let us explain this definition since the notation is somewhat non-standard. Let the sets A and B be subsets of $\mathbb{R}$ and have corresponding measures (not necessarily probabilistic) over these elements, F_A and F_B respectively. The notation $A \cup B$ represents a set of $\mathbb{R}$ with a measure $F_{A\cup B}$ that is the sum of these measures ($dF_{A\cup B} = dF_A + dF_B$). For example, suppose A is the set $[-10, 10]$ and B is $[0, 20]$ where F_A and F_B have a constant density 1 over these intervals. The set $A \cup B$ corresponds to the interval $[-10, 20]$ with measure $F_{A,B}$ that is twice as high on the interval $[0, 10]$ as compared to $[-10, 0]$ and $[10, 20]$. The expectation $E[g(y)|y \in S_{A,B}]$ is computed given the set $S_{A,B}$ by normalizing the corresponding measure of $S_{A,B}$ to be probabilistic. $B \cap \{y : g(y) < E[g(y)|y \in S_{A,B}]\}$ means that we are removing from set B all elements that are higher than the average y in $S_{A,B}$. When we remove elements from B, we do not change the measure of the remaining elements (so that the total measure of the set B drops by the measure of the removed elements): the re-normalization of measures happens only at the time when we compute the overall average. In this way, as we remove more and more elements from B, the overall average assigns higher and higher weight to the elements in A.

Here are some important properties of $S_{A,B}$:

Lemma 2 *Generalized Minimum Principle*

(0) $S_{A,B}$ *exists and is unique.*

(i) $E\left[g(y)|y \in A \cup B\right] \geq E\left[g(y)|y \in S_{A,B}\right]$, *with equality if and only if any* $y \in B$ *satisfies* $g(y) < E\left[g(y)|y \in S_{A,B}\right]$.

(ii) *Suppose that* $B' \supseteq B''$. *Then* $E[g(y)|y \in S_{A,B''}] \geq E[g(y)|y \in S_{A,B'}]$.

(iii) *Suppose that* $B' \supset B''$. *Then* $S_{A,B''} = S_{A,B'}$ *if and only if* $g(y) > E[g(y)|y \in S_{A,B''}]$ *for all* $y \in B' \backslash B''$.[9]

[8] Acharya et al. (2011) established a claim that is similar to (0) and (i) of the lemma below.

[9] Note that (ii) and (iii) imply that if there are elements $z \in B' \backslash B''$ such that $z < E\left[y|y \in S_{A,B''}\right]$ then $E\left[y|y \in S_{A,B''}\right] > E\left[y|y \in S_{A,B'}\right]$.

To see the intuition behind the existence of $S_{A,B}$ consider an iterative procedure of constructing $S_{A,B}$. In each step we remove from B some types that are higher than the previous average, which decreases the average. This is obviously a converging procedure, which stops at the latest after all types in B are removed (and then we are left with the set A to compute the expectation).

To see the intuition behind (*ii*) and (*iii*), take $g(y) = y$ and note that whether the expectation of Y conditional on $y \in S_{A,B}$ increases or decreases as we remove types from B depends on whether the removed types are higher or lower than the conditional average. In particular, when we exclude from B some y that are higher than the conditional average, this average does not change because these values are removed anyhow in the construction of $S_{A,B}$. However, when we remove from B realizations of y that are lower than the conditional average (even if these are above-average elements of B) then the conditional average goes up because these are below-average types in the original set $S_{A,B}$. The proof of (*ii*) and (*iii*) and the formalization of (0) and (*i*) are in the Appendix.

In our application, A corresponds to the set of uniformed agents and B corresponds to the set of potential disclosers. The function $g(y)$ corresponds to $P(x, y)$ given the revealed x. F_A is the probability distribution of Y conditional on $X = x$ times the ex-ante probability that the agent does not know Y, $(1-p)^2$. F_B is more complicated since B is a union of sets of potential disclosers that correspond to the different times that the agent could have learned Y and X, as we describe below. The plan of the proof is to compare the sets of potential disclosers after the two histories in question and apply Lemma 2 to establish the ranking of prices.

3.2 The Set of Potential Disclosers when X is Disclosed at t=1

In the next two subsections we describe in detail the sets of potential disclosers after the two histories . We start with the set after X has been revealed at $t = 1$. From Lemma 1 we know that at $t = 2$ the agent follows a myopic threshold policy. This implies that no disclosure of Y at $t = 2$ is bad news as compared to no disclosure of Y at $t = 1$ because investors know that the agent is informed with a higher probability at $t = 2$. Hence, the corresponding prices drop over time, i.e., $P_1(x, 1) \geq P_2(x, 1)$. As a result, an informed agent who has revealed X at $t = 1$ would reveal also Y if and only if it increases the current price (i.e., he follows a myopic strategy). The following lemma summarizes these observations:

Lemma 3 *In equilibrium, conditional on disclosure of X at $t = 1$:*

(i) if the agent does not reveal Y, prices drop over time, that is: $P_1(x,1) \geq P_2(x,1)$,

(ii) the agent's optimal disclosure strategy for Y is myopic at $t = 1$. That is, conditional on disclosing X at $t = 1$ an informed agent reveals also Y at $t = 1$ if and only if $P(x,y) \geq P_1(x,1)$.

If the agent revealed X at $t = 1$, investors know that $\tau_x = 1$ but there are ex-ante three possibilities regarding the time he learned Y, $\tau_y \in \{1,2,3\}$. We decompose the set of potential disclosers when X is disclosed at $t = 1$, B_1, into two disjoint subsets, $B_1 = B_1^1 \cup B_1^2$, based on when the agent has learned y. These subsets are given by:[10]

$$B_1^1 = \{y|\tau_x = \tau_y = 1,\ y \text{ is consistent with only } x \text{ being revealed at } t = 1\},$$

$$B_1^2 = \{y|\tau_x = 1, \tau_y = 2,\ y \text{ is consistent with } x \text{ being revealed at } t = 1\}.$$

Let A_1 denote the y coordinate of the set of uninformed agents. Before we remove from B_1^1 types which are not consistent with the public histories of potential disclosers, the sets A_1, B_1^1, B_1^2 have measures given by the conditional distribution of Y given $X = x$, multiplied by $\{(1-p)^2, p, p(1-p)\}$, respectively.

Using the notation introduced in the previous subsection, Lemma 1 implies that:

$$P_2(x,1) = E[P(x,y)|y \in S_{A_1,B_1}], \tag{1}$$

where

$$S_{A_1,B_1} \equiv A_1 \cup \{B_1 \cap \{y : P(x,y) < E[(P(x,y)|y \in S_{A_1,B_1})]\}\}.$$

If X is disclosed at $t = 1$ and Y was learned only at $t = 2$ then no realization of Y can be ruled out. Therefore, B_1^2 is the whole domain of Y.

On the other hand, B_1^1 can be described as the intersection of three conditions, $B_1^1 = C_1(x) \cap C_2(x) \cap C_3(x)$ where:

- $C_1(x)$: At $t = 1$, the agent prefers to reveal x instead of revealing both x and y. By Lemma 3, this condition is that y satisfies $P(x,y) \leq P_1(x,1)$.

- $C_2(x)$: At $t = 1$, the agent prefers to reveal x rather than y. Monotonicity of the equilibrium implies that this condition is $y \leq x$.

- $C_3(x)$: At $t = 1$, the agent prefers to reveal x rather than to hide both x and y.

[10] By "is consistent with" we mean the realizations of Y and τ_y that are consistent with the equilibrium path and the public histories $(x, t_x = 1)$ and $(x, y, t_x = 1, t_y = 2)$.

It is hard to fully pin down the equilibrium implications of the last condition. However, as the next lemma shows, if Theorem 1 did not hold, that is if $P_2(x,2) < P_2(x,1)$, we obtain a simple way to express equilibrium prices.

Lemma 4 *Suppose that in equilibrium $P_2(x,2) < P_2(x,1)$. Then, there exists $y^*(x) \geq Y_{x,2}$ and the corresponding set $B_1'^1 \equiv \{y|\tau_y = 1, y \leq \min\{x, y^*(x)\}\}$, such that if we replace B_1^1 with $B_1'^1$ in equation (1), the resulting price is still $P_2(x,1)$.*

3.3 The Set of Potential Disclosers when X is Disclosed at t=2

When X is disclosed at $t = 2$, investors in general do not know whether $\tau_x = 1$ or $\tau_x = 2$. This could make it more complicated to describe the price $P_2(x,2)$ since there would be four cases to consider for the potential disclosers: $(\tau_x, \tau_y) \in \{1,2\}^2$. However, as we prove in the Appendix, if Theorem 1 did not hold, that is if $P_2(x,2) < P_2(x,1)$, we could rule out that $\tau_x = 1$ if $t_x = 2$.

Lemma 5 *Suppose that in equilibrium $P_2(x,2) < P_2(x,1)$. Then the public history with $t_x = 2$ and $t_y > 1$ is consistent only with $\tau_x = 2$ (i.e., if the agent reveals X at $t = 2$, investors infer that the agent must have learned X at $t = 2$).*

The intuition is that the contradictory assumption, $P_2(x,2) < P_2(x,1)$, provides stronger incentives for an agent who has learned $X = x$ at $\tau_x = 1$ to disclose at $t_x = 1$ instead of waiting with disclosure till $t_x = 2$. The details of the proof are in the Appendix.

Since our proof of Theorem 1 is by contradiction, from now on we maintain the assumption that after the history $\hat{h}_2^P = (x,2)$ investors assign probability 1 to $\tau_x = 2$. This allows us to decompose the set of potential disclosers B_2 analogously to the decomposition of B_1 above. In particular, we decompose B_2 into two disjoint subsets, $B_2 = B_2^1 \cup B_2^2$:

$$B_2^1 = \{y|\tau_x = 2, \tau_y = 1,\ y \text{ is consistent with } x \text{ being revealed at } t = 2\},$$

$$B_2^2 = \{y|\tau_x = \tau_y = 2,\ y \text{ is consistent with } x \text{ being revealed at } t = 2\}.$$

The set A_2 is the y coordinate of the uniformed agents. The three sets have the same corresponding measures as in the case of A_1 and B_1.

Using the notation from Section 3.1 we can write:

$$P_2(x,2) = E[P(x,y)\,|y \in S_{A_2,B_2}], \tag{2}$$

where

$$S_{A_2,B_2} \equiv A_2 \cup \{B_2 \cap \{y : P(x,y) < E[(P(x,y)|y \in S_{A_2,B_2})]\}\}.$$

What realizations of Y are not consistent with equilibrium? First, for both sets, B_2^1 and B_2^2 we need to exclude types $y > x$ because, given our assumption that the equilibrium is symmetric and monotone, the agent would prefer to reveal y and not x in period 2 in those cases. This is in fact the only exclusion we can make in case of B_2^2, so $B_2^2 = \{y|\tau_x = \tau_y = 2, y \leq x\}$.[11]

Regarding B_2^1, we need to also exclude realizations of y that would have been disclosed at $t = 1$ if the agent knew only y at $t = 1$. Therefore: $B_2^1 = \{y|\tau_x = 2, \tau_y = 1, y \leq x, y \in ND\}$ where ND is the set of values of y that are not disclosed at $t = 1$ when the agent only knows y at $t = 1$.

3.4 Proof of the Main Theorem

Suppose by contradiction that $P_2(x,2) < P_2(x,1)$. Following Lemma 5 we can assume that the time when an agent discloses X coincides with when he has learned it, so that $t_x = \tau_x$. We divide the type space based on when the agent learns his information. Let L_{τ_x,τ_y} denote the set of types who learn X at τ_x and Y at τ_y (recall our convention that $\tau_y = 3$ means that the agent did not learn Y). In constructing the sets of uninformed and potential disclosers, we first condition on τ_x and then impose additional equilibrium conditions by removing certain types to obtain the set of potential disclosers.

When X is disclosed at $t = 1$, conditioning on $\tau_x = 1$ leads to $L_{1,1} \cup L_{1,2} \cup L_{1,3}$, where the set of uninformed, A_1, corresponds to the y coordinate of $L_{1,3}$ while the set of potential disclosers, B_1, corresponds to the y coordinate of a subset of $L_{1,1} \cup L_{1,2}$.

When X is disclosed at $t = 2$, conditioning on $\tau_x = 2$ leads to $L_{2,1} \cup L_{2,2} \cup L_{2,3}$ where the set of uninformed, A_2, corresponds to the y coordinate of $L_{2,3}$ while the set of potential disclosers, B_2, corresponds to the y coordinate of a subset of $L_{2,1} \cup L_{2,2}$.

When we condition on $\tau_x = 1$ the measure of $L_{1,3}$ is the same as the measure of $L_{2,3}$ when conditioning on $\tau_x = 2$. Since the conditional distribution of Y given X is independent of τ_x, when we project on the Y coordinate both cases lead to the same set; this implies that $A_1 = A_2$. In constructing B_1 and B_2 we start with $L_{1,1} \cup L_{1,2}$ and $L_{2,1} \cup L_{2,2}$, respectively. When we condition on $\tau_x = 1$ the measure of $L_{1,1} \cup L_{1,2}$ is the the same as measure of $L_{2,1} \cup L_{2,2}$ when conditioning

[11] We also know that y is such that the agent prefers to reveal x over keeping both x and y hidden. This can be ignored for computation of prices because it implies only that $P_2(\emptyset) \leq P_2(x,2)$, which is independent of y.

on $\tau_x = 2$. Since the conditional distribution of Y given X is independent of τ_x, when we project on the Y coordinate both cases lead to the same set.

We now eliminate types based on the equilibrium strategy to obtain the set of potential disclosers in both cases. From the previous two subsections, under the contradictory assumption that $P_2(x,2) < P_2(x,1)$, we can see that $B_1^2 \supseteq B_2^2$.

Regarding the comparison of $B_1'^1$ and B_2^1, define a set $B_2'^1 = \{y|\tau_x = 2, \tau_y = 1, y \leq \min\{x, y^*(x)\}, y \in ND\}$ where $y^*(x) > Y_{x,2}$ was introduced in Lemma 4 (in the definition of $B_1'^1$), so that $B_2'^1 \subseteq B_1'^1$. Using part (iii) of Lemma 2, if we replace B_2^1 with $B_2'^1$ in (2) then the resulting price is still $P_2(x,2)$ (because we are only adding to the set B_2^1 types above $P_2(x,2)$).

Combining these two comparisons, we get that $B_2' \subseteq B_1'$ for the sets used in the equations characterizing equilibrium prices, (1) and (2). This leads to a contradiction based on the Generalized Minimum Principle (Lemma 2): given the ranking of the sets of potential disclosers, it must be that $P_2(x,2) \geq P_2(x,1)$.

4 A Threshold Equilibrium

In this section we discuss additional properties of the equilibrium and then, under additional assumptions (linear payoff as well as normally distributed signals and V), we show (by construction of a threshold equilibrium) that the assumptions in Theorem 1 are non-vacuous and that for large enough values of x the inequality in the last part of the theorem is strict, so that later disclosures receive strictly better interpretation.

To see the difficulties in fully characterizing equilibria in our game, consider first a one-signal benchmark: the agent can only learn X and has a positive probability of learning it in any period. In equilibrium of that model the agent follows a myopic threshold strategy: he reports x if and only if it increases current price (and that price is by assumption increasing in the revealed signal). The reason is that the price upon non-disclosure is decreasing over time (as investors assign a higher and higher probability that the agent is informed) and hence there is no option value from waiting (for details see Acharya, DeMarzo, and Kremer (2011)).

It is quite different in our model. Consider an agent who learned only one signal, $X = x$. In period 2, assuming he has not disclosed anything yet, the agent is myopic and hence will disclose if and only if

$$P_2(x,2) \geq P_2(\emptyset). \tag{3}$$

In a model with only one signal, $P_2(x,2)$ is uniquely pinned down and is independent of the disclosure time. In contrast, in our model $P_2(x,2)$ depends on investors' beliefs about Y and these depend on the equilibrium strategy. It leads to two complications. First we have to specify off-equilibrium beliefs when the agent discloses a value of x that is off the equilibrium path (and the freedom to pick off-path beliefs leads to multiplicity of equilibria).[12] Second, even though $E[P(x,y)|X=x]$ is increasing in x, it does not guarantee that $P_2(x,2)$ is increasing because the disclosure policy for Y depends on the realized value of x (and depending on the realized x investors assign a different probability to the agent being informed about Y). If $P_2(x,2)$ were decreasing, agent's optimal strategy might fail to be a threshold one. A sufficient condition for prices to be increasing in x is that p is small because then the term $E[P(x,y)|X=x]$ corresponding to uninformed agents dominates.

The incentives to disclose for an agent who learned only one signal are even more complicated at $t=1$. That agent discloses $X=x$ if and only if:

$$\begin{aligned} & E_y\left[U\left(P_1(x,1), \max\left\{P(x,y), P_2(x,1)\right\}\right)|X=x\right] \\ & \geq E_y\left[U\left(P_1(\emptyset), \max\left\{P_2(\emptyset), P(x,y), P_2(x,2), P_2(y,2)\right\}\right)|X=x\right]. \end{aligned} \tag{4}$$

The left-hand side is the expected payoff if the agent discloses today, which takes into account that he may learn the other signal at $t=2$ and then decide to disclose it.[13] The right-hand side is the expected payoff if the agent decides not to reveal the signal today, which takes into account that in the next period he will have the option to either reveal nothing, reveal only x, or, if he learns the other signal in the meantime, he can reveal y or both signals. Condition (4) illustrates the main difficulty in constructing an equilibrium: both sides of the inequality depend on x. Even if all prices are increasing in x (which, as we discussed above, is not always guaranteed), whether agent's best response is a threshold strategy depends on whether the difference between the right-hand side and the left-hand side of (4) crosses zero only once; that in turn depends on the slopes of the different price functions and U.

Condition (4) also shows that a forward-looking agent benefits in two ways from delay of disclosure. First, he takes into account that he may learn $Y=y$ so large that

$$P_2(y,2) > \max\left\{P_2(\emptyset), P(x,y), P_2(x,2)\right\} \geq P_2(x,1),$$

[12] Multiplicity of equilibria can be also caused by multiple fixed points between the beliefs of the market and the agent's best response to these beliefs.

[13] We abuse notation here for brevity: the second-period maximization problem of the agent depends on whether he learns the second signal or not.

in which case he will disclose only the second signal at $t = 2$. Moreover, if the inequality in Theorem 1 is strict, i.e., $P_2(x, 1) < P_2(x, 2)$, then it creates a second benefit from delay of disclosure.[14] These benefits from delay imply that the equilibrium price for disclosure of a single signal has to be strictly higher than the non-disclosure price, i.e., $P_1(x, 1) > P_1(\emptyset)$ for all x disclosed on the equilibrium path. This is in contrast to equilibrium prices if the agent cares only about the first-period price or in a model with only one signal. For example, if the agent cared only about the first-period price (i.e., if $U(P_1, P_2)$ was constant in P_2), then the agent would reveal x in period 1 if and only if $P_1(x, 1) \geq P_1(\emptyset)$ and hence there would not need to be a uniform gap between disclosure and non-disclosure prices.[15] The more the agent cares about the second-period prices, the higher is his best-response threshold for disclosure (keeping the equilibrium prices fixed).

The strategy of the agent who knows both signals and has not yet revealed any of them is difficult to describe since it is a function of two variables and the incentives to disclose the higher signal depend on the value of the lower one (it is even difficult at $t = 2$ when the agent is myopic, as shown in Pae (2005)). Still, one property has to hold in equilibrium: suppose ND_t is the set of x such that if the agent knows only x at t he does not disclose it (in a threshold equilibrium, these are realizations below the threshold for an agent who knows one signal). Then, if the agent knows both x and y, in equilibrium he does not disclose only x if $x \in ND_t$ (but he could disclose either both or none of the signals). Otherwise, investors would infer that he must know the other signal and by an unraveling argument the agent would be better off disclosing both signals.

4.1 Normal Model

As this discussion illustrates, even in a two-signal, two-period model, the equilibrium conditions are quite complicated. To show existence of a well-behaved equilibrium we make additional assumptions.

Suppose that the value of the firm, V, is normally distributed and (without loss of generality), V has zero mean, i.e., $V \sim N(0, \sigma^2)$. The private signals that the manager may learn are given by $X = V + \tilde{\varepsilon}_x$ and $Y = V + \tilde{\varepsilon}_y$, where $\tilde{\varepsilon}_x, \tilde{\varepsilon}_y \sim N(0, \sigma_\varepsilon^2)$ and $\tilde{\varepsilon}_x$, $\tilde{\varepsilon}_y$ are independent of V and of each other. Finally, we assume that the manager maximizes sum of prices in two periods:

[14] Similar considerations appear when we analyze the incentives of an agent who knows both signals at $t = 1$ to disclose, since if he is planning not to disclose Y, he benefits from delaying disclosure because $P_2(x, 2) > P_2(x, 1)$. See the online appendix for details.

[15] If the agent cares only about the second-period price (i.e., if $U(P_1, P_2)$ is constant in P_1), there exists an equilibrium with no disclosure in the first period.

$U(P_1\left(H_1^P\right), P_2\left(H_2^P\right)) = P_1\left(H_1^P\right) + P_2\left(H_2^P\right)$.

The joint normal distribution of the signals implies the following conditional expectations:

$$E[V|X = x] = E[Y|X = x] = \beta_1 x,$$
$$P(x, y) = E[V|X = x, Y = y] = \beta_2(x + y),$$

where $\beta_1 = \frac{\sigma^2}{\sigma^2+\sigma_\varepsilon^2}$ and $\beta_2 = \frac{\sigma^2}{2\sigma^2+\sigma_\varepsilon^2}$.[16]

Next, we define a threshold strategy. Recall that we denote the agent's private information/history at $t = 2$ by $h_2^A \in H_2^A = \{(x, y, \tau_x, \tau_y)\}$. Without loss of generality, consider the case when the agent learns X either at $t = 1$ or $t = 2$, so that $\tau_x \leq 2$, and that $x \geq y$.

Definition 1 *Suppose that $h_2'^A$ and h_2^A are two private histories such that $h_2'^A$ differs from h_2^A only in the value of X, which equals x' under $h_2'^A$ and equals x under h_2^A. We say that a strategy is a threshold strategy if for any such $h_2'^A$ and h_2^A with $x' > x$ the following holds: if x is disclosed at time $t_x \in \{1, 2\}$ then x' is also disclosed at t_x or earlier. The equilibrium is a threshold equilibrium if the agent follows a threshold strategy.*

The following proposition states the main result of this section and is proven in the online appendix.

Proposition 1 *For $p < 0.77$ there exists a threshold equilibrium, characterized by a threshold x^*, in which:*

(i) an agent who at $t = 1$ learns only one signal discloses it at $t = 1$ if and only if it is greater than x^*. If the agent learns both signals at $t = 1$ and one of them is greater than x^* then he discloses at $t = 1$ either the highest signal or both signals. Disclosing a single signal $x < x^*$ at $t = 1$ is not part of the equilibrium disclosure strategy.

(ii) there exists $x' \geq x^*$ such that $P_2(x, 2) > P_2(x, 1)$ for any $x \geq x'$ and both public histories, $(x, 2), (x, 1)$, are on the equilibrium path.

(iii) for public histories on the equilibrium path, $P_t(x, t_x)$ is increasing in x (so the equilibrium is monotone).

[16] Note that $\beta_2 < \beta_1 < 2\beta_2 < 1$ and $\beta_2(1 + \beta_1) = \beta_1$. Also, note that $E[V|X = x] = \beta_1 x$ would be the price after disclosure of x if investors knew that the agent does not know Y. However, since investors assign a positive probability to the agent knowing Y and hiding it, equilibrium prices are lower and in general more complex.

In words, in our equilibrium there is a single threshold x^* such that if the agent knows only one of the signals at $t = 1$, he reveals it if and only if it is above x^* (there is also a threshold at $t = 2$ not described in the proposition). Moreover, if the agent knows both signals, he never discloses only one of them if it is lower than x^* (but he may disclose both). If at least one of the signals is above x^*, the agent reveals either one or both signals (and we do not rule out that the lower of the two revealed signals is below x^*).

The proof of Proposition 1 is complex and long, so it is delegated to the online appendix. The proof focuses on the incentives to disclose at $t = 1$ since this is where the dynamic considerations play a crucial role. The road-map of the proof is as follows. We first assume that the manager follows a threshold disclosure strategy. Then, for each public history we show properties of the equilibrium prices given *any* threshold strategy. In particular, using the assumption that by period 2 the agent learns signal Y with probability $p + p(1-p) < 0.95$ (which is implied by $p < 0.77$) we identify upper and lower bounds to the slopes of equilibrium prices $P_2(x, 2)$, $P_2(x, 1)$ and $P_1(x, 1)$ (as a function of x).[17] We use these bounds to show that the manager's expected payoff upon disclosure of a single signal is increasing faster in his signal as compared to his expected payoff upon non-disclosure (for example, that the left-hand side of (4) is increasing faster in x than the right-hand side, and similarly for all other private histories). This implies that it is indeed optimal for the manager to follow a threshold strategy in the first period, consistent with the initial assumption. We finish the proof by describing off-equilibrium beliefs and arguing that there exists a (fixed-point) threshold such that investors' beliefs and the agent's best response coincide.

5 Conclusions.

The vast literature on voluntary disclosure models focuses on static models in which an interested party (e.g., a firm's manager) may privately observe a single piece of private information (e.g., Dye 1985 and Jung and Kwon 1988). However, there are many real life circumstances in which, investors are uncertain about the time in which a firm observes value-relevant information and the disclosure of such information is voluntary. For instance, firms that have ongoing R&D projects can obtain new information about the state of their projects, where the time of information arrival and its content are unobservable to the market. Moreover, such information is not required to be publicly

[17] We conjecture that a threshold equilibrium exists also for values of p greater than 0.77; however, for tractability reasons we restrict the values of p since it simplifies the proof.

disclosed. One such example is pharmaceutical companies that get results of drug clinical trials. Investors' beliefs about a drug's clinical trial often have a great effect on the firm's price and may also affect investors' beliefs about the prospect of other projects of the firm. In such a setting, our model predicts that when the firm discloses the results of only part of its ongoing projects, a later disclosure gets a more positive market reaction (when keeping the disclosed information constant). Another related example is firms that apply for patents. After the initial application, the firm first waits to receive a notice of allowance (NOA) from the US PTO (U.S. Patent and Trademark Office) for each of the applications, which indicates that the patent is near approval. Typically, patent applications may include many claims to be covered under the patent and the NOA informs the firms which of the claims have been approved and which have not been approved. Following the NOA, the firm waits for the formal issuance, indicating that the PTO has formally bestowed patent protection.[18] As Lansford (2006, page 5) indicates: "It is important to note that firms enjoy wide discretion as to when to announce a patent event." Lansford (2006) documents that firms indeed time the disclosure of NOA strategically. In such circumstances, a manager deciding whether to disclose one piece of information must take into account the possibility of learning and potentially disclosing a new piece of information in the future. In this paper we have analyzed equilibrium consequences of such strategic considerations.

Our main result is that, in contrast to dynamic models with a single signal, the equilibrium reaction to voluntarily disclosed information depends not only on what is disclosed but also when, and that later disclosures receive a more favorable reaction even though the time the agent learns the signal is not informative per se.

Our discussion of condition (4) additionally suggests that the more the agent cares about the first-period prices (relative to the second-period prices) the more likely he should be to reveal information early. Multiplicity of equilibria makes it hard to precisely make/prove such a claim, but the intuition follows from (4): if we keep prices price functions as given, the agent's best response is to disclose a larger set of signals at $t = 1$ if he cares more about current prices. Higher weight assigned by the manager to the first period's price can reflect, for example, managers who face higher short-term incentives, managers of firms that are about to issue new debt or equity, a higher probability of the firm being taken over, a shorter expected horizon for the manager with the firm, etc. This intuition suggests a direction for new empirical investigation of how timing of

[18] It typically takes a few months between the NOA and the time at which the patent is published in the US PTO website.

voluntary disclosure by managers correlates with their long-term incentives.

6 Appendix

6.1 Later disclosure receives better responses

Proof of Lemma 2.

(0) For a constant c let $S^c_{A,B} \quad = A \cup \{B \cap \{(y, \tau_y) : g(y) \leq c\}$. For $c \to -\infty$ we have that $E\left[g(y)|y \in S^c_{A,B}\right] = E\left[g(y)|y \in A\right] > c$ and for $c \to \infty$ we have that $E\left[g(y)|y \in S^c_{A,B}\right] = E\left[g(y)|y \in A \cup B\right] < c$. From continuity we can find c^* for which $E_y\left[S^{c^*}_{A,B}\right] = c^*$. This establishes existence.

Now suppose by way of contradiction that there are multiple solutions. Specifically, assume there are $c' < c''$ so that $E\left[g(y)|y \in S^{c'}_{A,B}\right] = c'$ and $E\left[g(y)|y \in S^{c''}_{A,B}\right] = c''$. When we compare $S^{c'}_{A,B}$ to $S^{c''}_{A,B}$ we note that $S^{c''}_{A,B} \supset S^{c'}_{A,B}$ and that for $(y, \tau_y) \in S^{c''}_{A,B} \backslash S^{c'}_{A,B}$ we have $g(y) < E[g(y)|y \in S^{c''}_{A,B}]$. This implies that $S^{c''}_{A,B}$ can be represented as a union of $S^{c'}_{A,B}$, with the average $c' < c''$, and a set of types that are lower than c''. This however, implies that $E_y\left[S^{c''}_{A,B}\right] < c''$ and we get a contradiction.

(i) When comparing $S_{A,B}$ to $A \cup B$ we note that we have excluded above average types for which $g(y) > E\left[g(y)|y \in S_{A,B}\right]$. This results in lower average type.

(ii) Suppose first that there exists $(y, \tau_y) \in S_{A,B''} \backslash S_{A,B'}$. Since $B' \supseteq B''$ it must be that these $(y, \tau_y) \in B' \cap B''$. From the definition of $S_{A,B}$ since $(y, \tau_y) \in S_{A,B''}$ we conclude that $E\left[g(y)|y \in S_{A,B''}\right] > g(y)$. Since $(y, \tau_y) \notin S_{A,B'}$, we conclude that $E[g(y)|y \in S_{A,B'}] < g(y)$ which implies the claim. Hence, we will assume that $S_{A,B'} \supseteq S_{A,B''}$ and we consider $(y, \tau_y) \in S_{A,B'} \backslash S_{A,B''}$; this implies $y < E_y\left[S_{A,B'}\right]$. Hence, all the elements $(y, \tau_y) \in S_{A,B'} \backslash S_{A,B''}$ have y that is below the average in $S_{A,B'}$ which implies that $E_y\left[S_{A,B''}\right] \geq E_y\left[S_{A,B'}\right]$.

(iii) Consider the set $S_{A,B''}$, and note that it satisfy the definition for $S_{A,B'}$. Hence, the claim follows from uniqueness that was proven in (0). ■

Proof of Lemma 4. Since, $P_1(x, 1) \geq P_2(x, 1)$, we can apply part (iii) of Lemma 2 and ignore condition $C_1(x)$ in the definition of B'^1_1 because by doing that we only add types s.t. $P(x, y)$ is above the equilibrium price.

The constraint $C_3(x)$ can be described as $\Pi_x \geq \Pi_\emptyset$ where:

$$\Pi_\emptyset = U(P_1(\emptyset), \max\{P(x, y), P_2(x, 2), P_2(y, 2), P_2(\emptyset)\})$$

$$\Pi_x = U(P_1(x, 1), \max\{P(x, y), P_2(x, 1)\})$$

$\Pi_{\emptyset}$ is the expected payoff of a type that knows X and Y at time 1 and decides to reveal nothing; and Π_x is the payoff of the same type that decides to reveal x only. Since x is revealed alone on the equilibrium path at time $t = 1$, the inequality $\Pi_x \geq \Pi_{\emptyset}$ needs to hold. We also know that on the equilibrium path x is being disclosed at $t = 2$ which implies that $P_2(x, 2) \geq P_2(\emptyset)$. Condition $C_2(x)$ implies already that $y \leq x$ and by monotonicity of equilibrium $P_2(x, 2) \geq P_2(y, 2)$. So, without changing the intersection of $C_1(x) \cap C_2(x) \cap C_3(x)$ we can define $C_3(x)$ by replacing $\Pi_{\emptyset}$ with

$$\Pi'_{\emptyset} = U(P_1(\emptyset), \max\{P(x, y), P_2(x, 2)\}).$$

If $\Pi_x \geq \Pi'_{\emptyset}$ for all y then the constraint $C_3(x)$ can be ignored by defining $y^*(x) = \infty$. If this condition does not hold for any y then the agent does not disclose x at $t = 1$ if he knows both signals. This can be ruled out as an agent who only knows x decides to disclose it at $t = 1$. If for each realization of Y he would have preferred to keep quiet then this would be the case also when he does not know Y.

So we can focus on the case where $\Pi_x < \Pi'_{\emptyset}$ holds for some but not all y. Since we assumed $P_1(x, 1) > P_2(x, 2)$, this requires $P_1(x, 1) < P_1(\emptyset)$. In turn, that implies: (i) For y such that $P(x, y) > P_2(x, 1)$ the max in both Π_x and $\Pi'_{\emptyset}$ is attained at $P(x, y)$ and hence $\Pi_x < \Pi'_{\emptyset}$ in that range; (ii) For y such that $P(x, y) < P_2(x, 2)$ both Π_x and $\Pi'_{\emptyset}$ are independent of y and it has to be that $\Pi_x \geq \Pi'_{\emptyset}$ in that range since otherwise there would be no y for which $\Pi_x \geq \Pi'_{\emptyset}$; (iii) For y such that $P(x, y) \in [P_2(x, 2), P_2(x, 1)]$ we have that $\Pi'_{\emptyset}$ is constant while Π_x is increasing in y. Hence, there exists a unique $y^* \in [P_2(x, 2), P_2(x, 1)]$ for which $\Pi_x = \Pi'_{\emptyset}$. This y^* defines $C_3(x)$. ■

Proof of Lemma 5. We first rule out the possibility that the agent has learned X at $t = 1$ but not Y. Suppose by contradiction that in equilibrium the manager does not reveal $X = x$ at $t = 1$ when he knows only this signal (here we are using the restriction to pure strategy equilibria). Since, as we assumed, the public history $(x, 1)$ is on the equilibrium path, investors after that history would infer that the agent must know Y. The standard unraveling argument leads to a contradiction.

Next, we rule out the possibility that the agent learns both signals at $t = 1$. Let $\Pi_D(x)$ denote the payoff of an agent who knows only $X = x$ at $t = 1$ from disclosing x at $t = 1$; and let $\Pi_N(x)$ denote his payoff from not disclosing at $t = 1$. We have,

$$\Pi_D(x) = pE_y[\Pi_D(x, y)|X = x] + (1 - p)U(P_1(x, 1), P_2(x, 1))$$

$$\Pi_N(x) = pE_y[\Pi_N(x, y)|X = x] + (1 - p)U(P_1(\emptyset), \max\{P_2(x, 2), P_2(\emptyset)\})$$

where:

$$\Pi_D(x,y) = U(P_1(x,1), \max\{P_2(x,1), P(x,y)\}),$$

$$\Pi_N(x,y) = U(P_1(\emptyset), \max\{P_2(x,2), P_2(y,2), P(x,y), P_2(\emptyset)\}).$$

Since, as we argued in the beginning of this proof, if the agent knows only $X = x$ at $t = 1$ in equilibrium he discloses it, we have $\Pi_D(x) \geq \Pi_N(x)$. Consider now an agent who knows both signals at $t = 1$ and prefers to disclose just x at $t = 2$. Such an agent knows at time $t = 1$ that he will disclose x and not disclose y at $t = 2$. It must be that $\Pi'_N(x) \geq \Pi'_D(x)$ where:

$$\Pi'_D(x) = U(P_1(x,1), P_2(x,1))$$

$$\Pi'_N(x) = U(P_1(\emptyset), P_2(x,2)).$$

We claim that this leads to contradiction because $P_2(x,1) > P_2(x,2)$ implies that if $\Pi'_D(x) - \Pi'_N(x) \leq 0$ then $\Pi_D(x) - \Pi_N(x) < 0$.

To show this, note that $\Pi_D(x) - \Pi_N(x)$ is a weighted average over possible information sets of the agent in period 2. In case the agent does not learn Y in period 2, then trivially:

$$0 \geq \Pi'_D(x) - \Pi'_N(x) \geq U(P_1(x,1), P_2(x,1)) - U(P_1(\emptyset), \max\{P_2(x,2), P_2(\emptyset)\}).$$

For the harder case that the agent learns Y in period 2, start with the observations that for any increasing function U and any constants $\alpha_1, \alpha_2, \beta_1, \beta_2, \beta_3, \beta_4$, if:

$$U(\alpha_1, \beta_1) - U(\alpha_2, \beta_2) \leq 0$$
$$\text{and } \beta_1 > \beta_2$$

then

$$U(\alpha_1, \max\{\beta_1, \beta_3\}) - U(\alpha_2, \max\{\beta_2, \beta_3, \beta_4\}) \leq 0$$

and the inequality is strict if $\beta_3 > \beta_2$.

Applying it to our problem, we get that

$$\Pi'_D(x) - \Pi'_N(x) \leq 0$$
$$\text{and } P_2(x,1) > P_2(x,2)$$

implies that

$$\Pi_D(x,y) - \Pi_N(x,y) \leq 0$$

for all y and the inequality is strict for $P(x,y) > P_1(x,1)$. Taking the average over possible information sets in period 2 completes the reasoning. ■

7 Online Appendix: A Threshold Equilibrium

This Appendix is organized as follows. First, in Section 7.1, we discuss a variant of a static disclosure model that provides a numerical result and analytical insights we later use in the proof of Proposition 1. This variant of the static model may also be of independent interest. Then, in Section 7.2 we provide a proof of the Proposition. The proof starts with noticing properties of the equilibrium prices if the agent follows any threshold strategy. Given these properties, we show that the best response of the agent is indeed to follow a threshold strategy, establishing existence of a threshold equilibrium with the properties we discussed. In the same section we also establish the second claim in proposition 1. Finally, Section 7.3 contains omitted proofs of some lemmas describing the sensitivity of equilibrium prices to the disclosed signals if the agent follows a threshold strategy.

7.1 A Variant of a Static Model

Consider the following static disclosure setting, similar to Dye (1985) and Jung and Kwon (1988). With probability p the agent learns the firm's value, which is the realization of a random variable $S \sim N(\mu, \sigma^2)$.[19] If the agent learns the realization of S he may choose to disclose it. We are interested in investors' beliefs about the firm's value given no disclosure for an arbitrary threshold disclosure policy. That is, what is the expectation of S given that the agent discloses s if and only if $s \geq z$, for exogenously determined z. Unlike Dye (1985) and Jung and Kwon (1988), we are not constraining z to be consistent with optimal disclosure strategy by the agent, i.e., z is not part of an equilibrium. We will refer to this setting as the *"Dye setting with an exogenous disclosure threshold."*

Denote by $h^{stat}(\mu, z)$ investors' expectation of S given that no disclosure was made and given that the disclosure threshold is z. Figure 2 plots $h^{stat}(\mu, z)$ for $S \sim N(0, 1)$ and $p = 0.5$.

For $z \to \infty$ none of the agents discloses, and hence, following no disclosure investors do not revise their beliefs relative to the prior. For $z \to (-\infty)$ all agents who obtain a signal disclose it, and therefore, following no disclosure investors infer that the agent is uninformed, so investors posterior beliefs equal the prior distribution (as for $z \to \infty$). As the exogenous disclosure threshold, z, increases from $-\infty$, upon observing no disclosure investors know that the agent is either uninformed

[19] The reason we are considering general μ is that in our dynamic setting investors will update their beliefs about the undisclosed signal, y, based on the value of the disclosed signal, x.

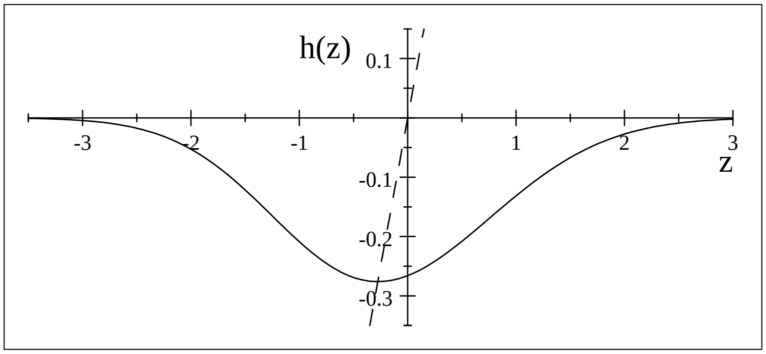

Figure 2: Price Given No-Disclosure in a Dye Setting with Exogenous Disclosure Threshold z

or that the agent is informed and his type is lower than z. Therefore, for any finite disclosure threshold, z, investors' expectation of S following no disclosure is lower than the prior mean (zero). The following lemma provides a further characterization of investors' expectation about S given no disclosure, $h^{stat}(\mu, z)$.

Lemma 6 *Consider the Dye setting with an exogenous disclosure threshold. Then:*

1. $h^{stat}(\mu+\Delta, z+\Delta) = h^{stat}(\mu, z) + \Delta$ *for any constant* Δ*; this implies that*

$$\frac{\partial}{\partial \mu} h^{stat}(\mu, z) + \frac{\partial}{\partial z} h^{stat}(\mu, z) = 1.$$

2. $z^* = \arg\min_z h^{stat}(\mu, z)$ *if and only if* $z^* = h^{stat}(\mu, z^*)$. *This implies that the equilibrium disclosure threshold in the standard Dye (1985) and Jung and Kwon (1988) equilibrium minimizes* $h^{stat}(\mu, z)$.

The second point follows from Lemma 2 (the Generalized Minimum Principle). Note that for all $z < h^{stat}(\mu, z)$ the price given no disclosure, $h^{stat}(\mu, z)$, is decreasing in z (and for $z > h^{stat}(\mu, z)$ it is increasing in z).

Direct analysis of the $h^{stat}(\mu, z)$ shows that:

Claim 1 (Numerical Result) *For* $p < 0.95$ *the absolute value of the slope of* $h^{stat}(\mu, z)$ *with respect to* z *is uniformly bounded by* 1.

We use this claim extensively in the proof below since it allow us to bound how future prices (in particular, $P_2(x, 1)$ and $P_2(x, 2)$) change with x and in turn that allows us to establish existence of a threshold strategy equilibrium. This is where we use the assumption $p < 0.77$ in the proof of Proposition 1. Note the difference in the bound in the proposition ($p < 0.77$) and in the claim ($p < 0.95$). The reason is that in the dynamic setup in period 2 the agent is informed about Y with probability $p + p(1-p)$, which needs to be less than 0.95 for us to apply this claim.

For the analysis of our dynamic model it will prove useful to consider an even richer variant of this model, allowing a random threshold policy. In particular, first, nature chooses publicly μ, the unconditional mean of S. Then, with probability λ_i, $i \in \{1, .., K\}$, where $\sum_{i=1}^{K} \lambda_i = p$, the agent discloses s if and only if $s \geq z_i(\mu)$.

The reason we are considering a random disclosure policy is as follows. In our dynamic setting, when by $t = 2$ the agent disclosed a single signal investors do not know whether the agent learned a second signal, and if so, whether he learned it at $t = 1$ or at $t = 2$. Since the agent follows different disclosure thresholds at the two possible dates, investors in equilibrium must assign a probability distribution over different disclosure thresholds. Moreover, in the dynamic model the disclosure thresholds for Y change with x and the disclosed x affects investors' unconditional expectation of Y. Therefore to apply these generic results to our dynamic model we write z as a function of the unconditional mean, μ.

Let us denote by $h^{stat}(\mu, \{z_i(\mu)\})$ the conditional expectation of S given no disclosure and given that the disclosure thresholds are $\{z_i(\mu)\}$ (assuming that $\{z_i(\mu)\}$ are differentiable).

Lemma 7 *For $p \leq 0.95$ suppose that $z_i(\mu) < h^{stat}(\mu, \{z_i(\mu)\})$ and $z_i'(\mu) \in [0, c]$ for all i. Then $\frac{d}{d\mu} h^{stat}(\mu, \{z_i(\mu)\}) \in (\min\{1, 2-c\}, 2)$.*

Before we formally prove Lemma 7, we analyze the particular case in which the disclosure strategy is nonrandom, i.e., $K = 1$. This provides the basic intuition for Lemma 7.

We start by providing the two simplest examples, for the cases where $z'(\mu) = 1$ and $z_i'(\mu) = 0$. These examples are useful in demonstrating the basic logic and how it can be analyzed using Figure 2. These two examples also provide most of the intuition for the case with no restriction on $z_i'(\mu)$, which is presented in Example 3. Note that Example 3 also provides the upper and lower bounds for the more general case in Lemma 7.

Examples (all the examples assume $K = 1$):

1. If $z'(\mu) = 1$ then $\frac{d}{d\mu} h^{stat}(\mu, z(\mu)) = 1$.

   Using point 1 in Lemma 6 we have $\frac{d}{d\mu} h^{stat}(\mu, z(\mu)) = \frac{\partial}{\partial\mu} h^{stat}(\mu, z) + z'(\mu) * \frac{\partial}{\partial z} h^{stat}(\mu, z) = 1$.

   The intuition can be demonstrated using Figure 2. A unit increase in μ (keeping z constant) shifts the entire graph both upwards and to the right by one unit. However, since also z increases by one unit, the overall effect is an increase in $h^{stat}(\mu, z(\mu))$ by one unit.

2. If $z'(\mu) = 0$ and $z(\mu) = z^*$, then $\frac{d}{d\mu} h^{stat}(\mu, z(\mu)) \in (1, 2)$.

   From Lemma 6 we know that $\frac{\partial}{\partial\mu} h^{stat}(\mu, z^*) + \frac{\partial}{\partial z^*} h^{stat}(\mu, z^*) = 1$ and therefore $\frac{\partial}{\partial\mu} h^{stat}(\mu, z^*) = 1 - \frac{\partial}{\partial z^*} h^{stat}(\mu, z^*)$. From Claim 1 we also know that $\frac{\partial}{\partial z^*} h^{stat}(\mu, z^*) \in (-1, 0)$ since $z^* \leq h^{stat}(\mu, z^*)$. Therefore, $\frac{\partial}{\partial\mu} h^{stat}(\mu, z^*) \in (1, 2)$. The intuition can be demonstrated using Figure 2. The effect of a unit increase in μ can be presented as a sum of two effects: (i) a unit increase in the disclosure threshold, z, as well as a shift of the entire graph both to the right and upwards by one unit, and (ii) a unit decrease in the disclosure threshold, z, (as $z'(\mu) = 0$). The first effect is similar to Example 1 above and therefore increases $h^{stat}(\mu, z(\mu))$ by one. The second effect increases $h^{stat}(\mu, z(\mu))$ by the absolute value of the slope of $h^{stat}(\mu, z)$, which is between zero and one.

3. In case $z'(\mu) = c$, we have $\frac{d}{d\mu} h^{stat}(\mu, z(\mu)) \in (\min\{1, 2-c\}, \max\{1, 2-c\})$.

   The previous examples are nested in this more general case. Following a similar logic, we conclude that $\frac{d}{d\mu} h^{stat}(\mu, z(\mu)) = \frac{\partial}{\partial\mu} h^{stat}(\mu, z) + c\frac{\partial}{\partial z} h^{stat}(\mu, z) = 1 + (c-1)\frac{\partial}{\partial z} h^{stat}(\mu, z(\mu))$. Recall that $\frac{\partial}{\partial z} h^{stat}(\mu, z(\mu)) \in (-1, 0)$ for $p < 0.95$.

We next provide the a formal proof of Lemma 7.

Proof of Lemma 7

By applying Bayes role, $h^{stat}(\mu, \{z_i(\mu)\})$ is given by:

$$h^{stat}(\mu, \{z_i(\mu)\}) = \frac{(1-p)\mu + \sum_{i=1}^{K} \lambda_i \int_{-\infty}^{z_i(\mu)} y\phi(y|\mu)\, dy}{(1-p) + \sum_{i=1}^{K} \lambda_i \Phi(z_i(\mu)|\mu)}.$$

Taking the derivative of $h^{stat}(\mu, \{z_i(\mu)\})$ with respect to μ and applying some algebraic manipulation yields:

$$\frac{d}{d\mu} h^{stat}(\mu, \{z_i(\mu)\}) = 1 + \frac{\sum_{i=1}^{K} \lambda_i (z_i'(\mu) - 1)\phi(z_i(\mu)|\mu)\left(z_i(\mu) - h^{stat}(\mu, \{z_i(\mu)\})\right)}{(1-p) + \sum_{i=1}^{K} \lambda_i \Phi(z_i(\mu)|\mu)}.$$

We start by proving the supremum of this derivative.

Given that $z_i'(\mu) \geq 0$ and $z_i(\mu) \leq h^{stat}(\mu, \{z_i(\mu)\})$ for all $i \in \{1, .., K\}$ we have

$$\begin{aligned}\frac{d}{d\mu} h^{stat}(\mu, \{z_i(\mu)\}) &\leq 1 + \frac{\sum_{i=1}^{K} \lambda_i \phi(z_i(\mu)|\mu)\left(h^{stat}(\mu, \{z_i(\mu)\}) - z_i(\mu)\right)}{(1-p) + \sum_{i=1}^{K} \lambda_i \Phi(z_i(\mu)|\mu)} \\ &\leq 1 + \max_{\substack{z_i \leq h(x) \\ i \in \{1,...K\}}} \frac{\sum_{i=1}^{K} \lambda_i \phi(z_i(\mu)|\mu)\left(h^{stat}(\mu, \{z_i(\mu)\}) - z_i(\mu)\right)}{(1-p) + \sum_{i=1}^{K} \lambda_i \Phi(z_i(\mu)|\mu)}.\end{aligned} \tag{5}$$

Due to symmetry, for all $i \in \{1, .., K\}$ the maximum is achieved at the same $z_i(\mu) = \hat{z}(\mu)$. To see this, note that the FOC of the maximization with respect to $z_i(\mu)$ is

$$\begin{aligned}0 &= \left(\phi'(z_i(\mu)|\mu)\left(h^{stat}(\mu, \{z_i(\mu)\}) - z_i(\mu)\right) - \phi(z_i(\mu)|\mu)\right)\left((1-p) + \sum_{i=1}^{K} \lambda_i \Phi(z_i(\mu)|\mu)\right) \\ &- \left(\sum_{i=1}^{K} \lambda_i \phi(z_i(\mu)|\mu)\left(h^{stat}(\mu, \{z_i(\mu)\}) - z_i(\mu)\right)\right)\phi(z_i(\mu)|\mu).\end{aligned}$$

Since $\phi'(z_i(\mu)|\mu) = -\alpha(z_i(\mu) - \mu)\phi(z_i(\mu)|\mu)$ (for some constant $\alpha > 0$), this simplifies to

$$-\alpha(z_i(\mu) - \mu)\left(h^{stat}(\mu, \{z_i(\mu)\}) - z_i(\mu)\right) = \frac{\sum_{i=1}^{K} \lambda_i \phi(z_i(\mu)|\mu)\left(h^{stat}(\mu, \{z_i(\mu)\}) - z_i(\mu)\right)}{(1-p) + \sum_{i=1}^{K} \lambda_i \Phi(z_i(\mu)|\mu)} + 1.$$

In the range $z_i(\mu) \leq h^{stat}(\mu, \{z_i(\mu)\}) \leq \mu$, the LHS is decreasing in $z_i(\mu)$.[20] The RHS is the same for all i. Therefore, the unique solution to this system of FOC is for all $z_i(\mu)$ to be equal (and note that the maximum is achieved at an interior point since at $z_i(\mu) = h^{stat}(\mu, \{z_i(\mu)\})$ the LHS is zero and the RHS is positive; and as $z_i(\mu)$ goes to $-\infty$ the LHS goes to $+\infty$ while the RHS is bounded).

Returning to the bound in (5), that the maximum is achieved for some $\hat{z}(\mu)$ constant for all i, implies that Example 3 (discussed above) can be used to provide the upper bound: $\frac{d}{d\mu} h^{stat}(\mu, \{z_i(\mu)\}) \leq \max\{1, 2 - \min_i\{z_i'(\mu)\}\}$. The lower bound can be achieved in a similar way by observing that if we want to minimize the slope we will again choose the same $z_i(\mu)$ for all i, and therefore by Example 3 $\frac{d}{d\mu} h^{stat}(\mu, \{z_i(\mu)\}) \geq \min\{1, 2 - \max_i\{z_i'(\mu)\}\}$.[21] Computing uniform bounds over all slopes $z_i'(\mu) \in [0, c]$ yields the result.

QED Lemma 7

7.2 Existence of a Threshold Equilibrium

We now turn the proof of existence of a threshold equilibrium. The proof of Proposition 1 is complicated and technical, so we start with a road-map.

[20] Since $z_i(x) \leq h(x, \{z_i(\cdot)\})$ also $h(x, \{z_i(\cdot)\}) \leq E[x|y] = \beta_1 x$.

[21] For a complete analysis of this case see proof of Lemma 12 below.

Road-map of Proof of Proposition 1

First, we assume that the manager follows some threshold strategy and establish bounds on the slopes of equilibrium prices under the assumption that $p < 0.77$ (Claim 2 below). We then show that if prices have these properties then the manager's best response is indeed to follow a threshold strategy. This requires looking at all possible private histories of the agent and verifying that claim for each one of them. By appropriately choosing off-equilibrium beliefs, we then establish the existence of a threshold equilibrium. Finally, in the last step of the proof we show that there exists an x' such that for $x > x'$ later disclosure receives a strictly better interpretation, i.e., $P_2(x, 2) > P_2(x, 1)$. To keep the flow of the reasoning we delegate some of the most algebra-heavy proofs to subsection 7.3.

Proof of Proposition 1

To establish existence of a threshold equilibrium we need to look at many possible private histories at $t = 1$ and $t = 2$. We make the following observations about all equilibria:

1) Once an agent reveals one of the signals, he follows a myopic disclosure strategy (i.e. reveals the second signal if and only if it increases the current price), so his disclosure policy is a threshold policy (see Lemma 3).

2) At $t = 2$, if the agent has not revealed any of the signals, he reveals at least one if $P_2(\emptyset) \leq P_2(x, 2)$. For this to be a threshold strategy we need that $P_2(x, 2)$ is increasing (as in Pae (2005)). We establish this property below for the equilibria we construct.

3) In a threshold equilibrium we must have that at $t = 1$, $P_1(\emptyset) < P_1(x, 1)$ for any $x \geq x^*$. Otherwise an agent that leaned only the signal X at $t = 1$ would strictly prefer to postpone disclosure since there is a positive probability that he will learn the second signal at $t = 2$ and reveal only the second signal (if $P_2(y, 2) > \max\{P(x, y), P_2(x, 2), P_2(\emptyset)\}$). In addition, recall that $P_2(x, 2) \geq P_2(x, 1)$ so there may be another benefit to waiting, which applies to both the agent that learned on X at $t = 1$ and for an agent that learned both signals at $t = 1$ and will disclose only one signal by $t = 2$.

4) The most difficult analysis is for $t = 1$ since the agent incentives to disclose depend not only on the current prices but also on how his current disclosure affects continuation payoffs. Therefore, most of our proof considers different possible private histories of the agent at $t = 1$.

It proves convenient to introduce a new definition:

Definition 2 *Denote investors' expectation of the value of the signal y, as of time t, given that* *the manager disclosed only x at time t_x, by $h_t(x, t_x)$. The notation is borrowed from the notation of prices, $P_t(x, t_x)$.*

With this notation, the equilibrium prices that play a central role in our proof are:

$$P_1(x, 1) = \beta_2 (x + h_1(x, 1)),$$
$$P_2(x, 1) = \beta_2 (x + h_2(x, 1)),$$
$$P_2(x, 2) = \beta_2 (x + h_2(x, 2)).$$

The following Claim derives upper and lower bounds to the slopes of these prices:

Claim 2 *Suppose that investors believe that the manager follows a threshold reporting strategy as in Proposition 1. Then, for $p \leq 0.77$ and $x > x^*$:*

$$\frac{\partial}{\partial x} h_1(x, 1) \begin{cases} = \beta_1 & if \quad h_1(x, 1) < x \\ \in (2\beta_1 - 1, \beta_1) & if \quad h_1(x, 1) > x \end{cases},$$

$$\frac{\partial}{\partial x} h_2(x, 2) \begin{cases} = \beta_1 & if \quad h_2(x, 2) < x^* \\ \in (2\beta_1 - 1, 2\beta_1) & if \quad h_2(x, 2) > x^* \end{cases}, \tag{6}$$

$$\frac{\partial}{\partial x} h_2(x, 1) \begin{cases} = \beta_1 & if \quad h_2(x, 1) < x \\ \in (2\beta_1 - 1, \beta_1) & if \quad h_2(x, 1) > x \end{cases}.$$

This Claim established part (iii) of the Proposition. In particular, the bound on $\frac{\partial}{\partial x} h_2(x, 2)$ implies that $P_2(x, 2)$ increases in x, so the agent indeed best responds with a threshold strategy at time $t = 2$. So from now on we focus on $t = 1$.

In proving the existence of a threshold equilibrium, we first consider partially informed agents that learn a single signal, x, at $t = 1$ ($\tau_x = 1, \tau_y \neq 1$) and then we consider fully informed agents that learn both signals at $t = 1$. For each of these cases we show that: (i) for sufficiently high (low) realizations of x the agent discloses (does not disclose) x at $t = 1$; and (ii) On the equilibrium path, the difference between the agent's expected payoff if he discloses only x at $t = 1$ and if he does not disclose at $t = 1$ is increasing in x, implying that the agent's best response is indeed a threshold strategy.

Partially Informed Agents ($\tau_x = 1, \tau_y \neq 1$)

First consider an agent that knows only x at $t = 1$. For sufficiently low realizations of x the agent is always better off not disclosing it at $t = 1$, as he can "hide" behind uninformed agents.

We next establish that for an agent that learns a single signal, x, at $t=1$ his incentives to disclose it are monotone in x and hence a threshold strategy is a best response (the proof of the lemma is in the next subsection).

Lemma 8 *Consider an agent that learns a single signal, x, at $t=1$. If $\beta_1 \geq \frac{1}{2}$ or if $h_2(x^*,1) \leq x^*$ then if investors believe that the agent follows a threshold strategy, the incentives to disclose x at $t=1$ are strictly increasing in x. That is,*

$$\frac{\partial}{\partial x}\left(E\left(U|\tau_x=1,\tau_y\neq 1,t_x=1\right)-E\left(U|\tau_x=1,\tau_y\neq 1,t_x\neq 1\right)\right)>0,$$

and there exists x high enough that the agent is better off revealing it than not.

Fully Informed Agents ($\tau_x=\tau_y=1$)

We next discuss an agent that learns both signals at $t=1$ (such that $x>y$).

Using Theorem 1, we divide these private histories into three cases:

1) $y \geq h_2(x,2)$.

2) $y \in (h_2(x,1), h_2(x,2))$

3) $y \leq h_2(x,1)$

Types $y \geq h_2(x,2)$ disclose y in period 2 no matter if they disclose x at $t=1$ or not. Therefore, such types will disclose x at $t=1$ if

$$\max\{P_1(x,1), P(x,y)\} \geq P_1(\emptyset).$$

and since Claim 2 implies that the left-hand side is increasing in x, these types will follow a threshold strategy.

Now consider the case $y \leq h_1(x,1)$ so that y is sufficiently low that it will not be disclosed at $t=2$ if it was not disclosed at $t=1$ but the agent disclosed x. There are two sub-cases: either after not disclosing x at $t=1$ the agent will remain silent at $t=2$ or he will disclose x. The first sub-case is easier since the payoff from non-disclosing x is a constant and hence the incentives to disclose are increasing in x if and only if $P_1(x,1)+P_2(x,1)$ are increasing and that follows from Claim 2. The next lemma covers the second sub-case.

Lemma 9 *Consider an agent that learned both signals at $t=1$ and the realization of $y \leq x$ is such that $y \leq h_2(x,1)$ (so that it will not be disclosed at $t=2$) and $\beta_2[x+h_2(x,2)] \geq P_2(\emptyset)$. Then:*

(i) *For sufficiently high realizations of x the agent prefers to disclose x at $t = 1$ over not disclosing x at $t = 1$.*

(ii) $\frac{\partial}{\partial x}(E(U|\tau_x = 1, \tau_y = 1, t_x = 1) - E(U|\tau_x = 1, \tau_y = 1, t_x \neq 1)) > 0.$

Proof.

(i) We need to show that for sufficiently high x :

$$\beta_2 [x + h_1 (x, 1)] + \beta_2 [x + h_2 (x, 1)] > P_1 (\emptyset) + \beta_2 [x + h_2 (x, 2)] .$$

Rearranging yields

$$\beta_2 [x + h_2 (x, 1)] - P_1 (\emptyset) > \beta_2 [h_2 (x, 2) - h_1 (x, 1)] .$$

By Claim 2 the LHS of the above inequality, $\beta_2 [x + h_2 (x, 1)] - P_1 (\emptyset)$, goes to infinity as x goes to infinity. Therefore, it is sufficient to show that $h_2 (x, 2) - h_1 (x, 1)$ is bounded from above. Both $h_2 (x, 2)$ and $h_1 (x, 1)$ are lower than $\beta_2 x$. From the Generalized Minimum Principle (Lemma 2) we know that $h_1 (x, 1)$ is higher than the price given no disclosure in a Dye (1985), Jung and Kwon (1988) setting where $y \sim N (\beta_1 x, Var (y|x))$. The price given no disclosure in such a setting is $\beta_1 x - Const$, so $h_1 (x, 1) > \beta_1 x - Const$. Hence, given that $h_2 (x, 2) < \beta_1 x$ we have $h_2 (x, 2) - h_1 (x, 1) < Const$.

(ii) We need to show that

$$\frac{\partial}{\partial x} (\beta_2 [x + h_1 (x, 1)] + \beta_2 [x + h_2 (x, 1)] - P_1 (\emptyset) - \beta_2 [x + h_2 (x, 2)]) > 0,$$

which is identical to condition 2 in the proof of Lemma 8. ■

Finally, for the sub-case $y \in (h_2 (x, 1) , h_2 (x, 2))$ the agent will reveal y in period 2 if he reveals x at $t = 1$, but will not reveal it if he does not reveal x at $t = 1$. This agent will reveal x today if

$$\beta_2 [x + h_1 (x, 1)] + \beta_2 [x + y] > P_1 (\emptyset) + \beta_2 [x + h_2 (x, 2)] .$$

and these incentives are monotone in x for the same reasons as in the previous lemma.

Fixed point and off-equilibrium beliefs

That finishes the analysis of all possible private histories. To summarize, we have proven that (assuming $p < 0.77$) if $\beta_2 \geq \frac{1}{2}$ or if or if $h_2 (x^*, 1) \leq x^*$, then the best response of the agent is to indeed follow a threshold strategy. We now need to find a fixed-point for the threshold. That is, we need to find x^* such that if the market believes that in period 1 the agent uses threshold x^* then he best responds using that exact threshold. We also need to specify off-equilibrium beliefs and it turns out that these two tasks are connected.

In a model with only one signal (static or dynamic), the only off-path history is when the agent reveals a signal below the equilibrium threshold but that does not matter for beliefs since at that point there is no information asymmetry. In contrast, in a model with two signals, when the agent reveals only one of them and it is below x^*, we need to specify the market's beliefs about the probability that he has learned the other signal and if so, what is y. In particular, we can set the beliefs to be arbitrarily negative about y and hence the price $P_t(x, t_x)$ to be arbitrarily low off-path, making sure that the agent does not have incentives to reveal such x.

Therefore, any x^* such that for all $x \geq x^*$ the agent prefers (weakly or strictly) to reveal x (and possibly also y) when he is partially informed (knows only x) or fully informed (knows both x and y, in which case the incentives have to hold for all $y \leq x$) can be used to complete a construction of our equilibrium. (Note: a model with two-dimensional signals has multiple equilibria supported by appropriate off-path beliefs).

To see that such x^* exists note that as investors belief x^* goes to infinity then the price upon nondisclosure, $P_1(\emptyset)$ converges to 0 (since there is no inference from nondisclosure in the limit) while for any $x > x^*$ prices $P_1(x, 1)$ and $P_2(x, 1)$ get arbitrarily large (and recall that we have proven above that $h_2(x, 2) - h_1(x, 1) < Const$). So for sufficiently large x^* after all private histories in period 1 the agent prefers to reveal x if it is above x^* to not revealing anything.

That finishes the proof that there exists an equilibrium in threshold strategies.

Finally, we establish in the following Lemma the last part of Proposition 1.

Lemma 10 *There exists an $x' \geq x^*$ such that $P_2(x, 2) > P_2(x, 1)$ for any $x \geq x'$.*

Proof. In Theorem 1 we have shown that $P_2(x, 2) \geq P_2(x, 1)$ for any x, which implies in the setting of section 4 that $h_2(x, 2) \geq h_2(x, 1)$.

As established in section 3, given disclosure of the signal x the manager behaves myopically in the sense that he discloses the signal y (when he learned y) if and only if it increases the price relative to the price when y is not disclosed. This holds for both $t = 1$ and $t = 2$. We can now introduce the equilibrium inference on the sets B_1^1, B_1^2, B_2^1 and B_2^2 that were defined in section 3.

In particular, we adjust the set B_i^j by taking into account also the equilibrium disclosure strategy when defining the potential disclosers and denote it by b_i^j. The sets b_i^j for $i,j=1,2$ are given by:

$$b_1^1 = \{(y,\tau_y)|\tau_y = 1,\ t_x = 1 \text{ and } y \leq \min\{x, h_1(x,1), h_2(x,1)\}\}$$
$$b_1^2 = \{(y,\tau_y)|\tau_y = 2,\ t_x = 1 \text{ and } y \leq h_2(x,2)\}$$

$$b_2^1 = \{(y,\tau_y)|\tau_y = 1,\ t_x = 2 \text{ and } y \leq \min\{x^*, h_2(x,2)\}\}$$
$$b_2^2 = \{(y,\tau_y)|\tau_y = 2,\ t_x = 2 \text{ and } y \leq \min\{x, h_2(x,2)\}\}$$

Note that $h_1(x,1) > h_2(x,1)$ so b_1^1 can be written as $b_1^1 = \{(y,\tau_y)|\tau_y = 1,\ t_x = 1 \text{ and } y \leq \min\{x, h_2(x,1)\}\}$.

We next show that $h_2(x,2) > h_2(x,1)$ for all x such that $h_2(x,2) > x^*$. From section 3 we know that $h_2(x,2) \geq h_2(x,1)$ so we only need to preclude $h_2(x,2) = h_2(x,1)$. Assume by contradiction that $h_2(x,2) = h_2(x,1)$. Since $x > x^*$ we have $b_2^1 \subset b_1^1$ and $b_2^2 \subseteq b_1^2$. Moreover, any $y \in (x^*, h_2(x,2))$ is strictly lower than $h_2(x,2)$ which equals $E[y|y \in S_{A,b_2}]$. From part (i) of the Generalized Minimum Principle (Lemma 2) we have $h_2(x,2) > h_2(x,1)$ which leads to a contradiction. Therefore, for all values of x such that $h_2(x,2) > x^*$ we have $h_2(x,2) > h_2(x,1)$.

The last thing to be shown is that there exists an x' such that $h_2(x,2) > x^*$ for any $x \geq x'$. This is immediate given that $\frac{\partial}{\partial x} h_2(x,2) = \beta_1 (> 0)$ for value of x such that $h_2(x,2) < x^*$ (see Claim 2). Note that x' can be, but is not necessarily, greater than x^*. ■

QED Proposition 1.

7.3 Omitted Proofs.

In this section we provide proofs for the lemmas and claims in the previous section.

7.3.1 Proof of Claim 2

Claim 2 above is:

Suppose that investors believe that the manager follows a threshold reporting strategy as in

Proposition 1. Then, for $p \leq 0.77$:

$$\frac{\partial}{\partial x}h_1(x,1)\begin{cases} =\beta_1 & if \quad h_1(x,1)<x \\ \in(2\beta_1-1,\beta_1) & if \quad h_1(x,1)>x \end{cases},$$

$$\frac{\partial}{\partial x}h_2(x,2)\begin{cases} =\beta_1 & if \quad h_2(x,2)<x^* \\ \in(2\beta_1-1,2\beta_1) & if \quad h_2(x,2)>x^* \end{cases}, \tag{7}$$

$$\frac{\partial}{\partial x}h_2(x,1)\begin{cases} =\beta_1 & if \quad h_2(x,1)<x \\ \in(2\beta_1-1,\beta_1) & if \quad h_2(x,1)>x \end{cases}.$$

Proof of Claim 2

In the proof we use a terminology "non-binding" and "binding" case to distinguish between $h_t(x,1) \leq x$ and $h_t(x,1) > x$. These cases are qualitatively different because in general investors infer that if $\tau_y = 1$ then $y \leq x$ and $y < h_t(x,1)$. In the "non-binding" the second inequality implies the first. In the "binding" case, $y \leq x$ implies the second inequality.

We start the proof with $\frac{\partial}{\partial x}h_1(x,1)$.

Lemma 11 *For $p \leq 0.95$, $\frac{\partial}{\partial x}h_1(x,1)\begin{cases} =\beta_1 & if \quad h_1(x,1)<x \\ \in(2\beta_1-1,\beta_1) & if \quad h_1(x,1)>x \end{cases}$*

Proof. As shown in Section 3, for any x that is disclosed at $t=1$ such that $h_1(x,1) < x$ (the non binding case), if $\tau_y = 1$ the agent is myopic with respect to the disclosure of y and discloses it whenever $y \geq h_1(x,1)$. This case is captured by Example 1 in Section 7.1: an increase in the mean of the distribution results in an identical increase in both the equilibrium beliefs and the equilibrium disclosure threshold. The quantitative difference in our setting is that a unit increase in x increases investors' beliefs about y by β_1 (rather than by 1), and therefore also increases both the beliefs about y and the threshold for disclosure of y by β_1. As a result, for $h_1(x,1) < x$ we have $\frac{\partial}{\partial x}h_1(x,1) = \beta_1$.[22]

In the binding case, i.e., for all x such that $h_1(x,1) > x$ (if such $x > x^*$ exists) we know that if $\tau_y = 1$ then $y < x$ (otherwise, the manager would have disclosed y). An increase in x increases the beliefs about y at a rate of β_1, while the increase in the constraint/disclosure threshold ($y < x$) increases the beliefs about y at a rate of 1. Therefore, this is a special case of Example 3 in Section

[22] Since both the beliefs about Y and the disclosure threshold increase at the same rate, the probability that the agent learned y at $t=1$ but did not disclose it, conditional on him disclosing x at $t=1$, is independent of x.

7.1, where we increase the mean by β_1 and $z'(\mu) \equiv c = \frac{1}{\beta_1}$. From Example 3 we know that an increase in the beliefs about y given a unit increase in x (which is equivalent to an increase of β_1 in the value of μ in Example 3) is given by $\beta_1 \left(1 + (c-1) \frac{\partial}{\partial z} h^{stat}(\mu, z)\right)$. Substituting $c = \frac{1}{\beta_1}$ and rearranging terms yields

$$\frac{\partial}{\partial x} h_1(x,1) = \beta_1 + (1-\beta_1) \frac{\partial}{\partial z} h^{stat}(\mu, z).$$

Since $\frac{\partial}{\partial z} h^{stat}(\mu, z) \in (-1, 0)$ (recall Claim 1), we have $\frac{\partial}{\partial x} h_1(x,1) \in (2\beta_1 - 1, \beta_1)$. ■

Analyzing the effect of x on $h_2(x,2)$ and $h_2(x,1)$ is more involved and more technical. The reason these cases are more complicated is that when pricing the firm at $t = 2$ investors do not know whether the manager learned y at $t = 1$ or at $t = 2$ (in the case where the agent did in fact learn y). Investors' inferences about y depend on when the agent learned it, and therefore the analysis of $h_2(x,2)$ and $h_2(x,1)$ requires stochastic disclosure thresholds. This is where we use Lemma 7.

We next analyze $h_2(x,2)$.

When an agent discloses $x > x^*$ at $t = 2$ investors know that $\tau_x = 2$ (otherwise the agent would have disclosed x at $t = 1$). Investors' beliefs about the manager's other signal at $t = 2$ is set as a weighted average of three scenarios: $\tau_y = 1$, $\tau_y = 2$ and $\tau_y > 2$. We start by describing the disclosure thresholds conditional on each of the three scenarios.

(i) If $\tau_y > 2$ the agent cannot disclose y and therefore the disclosure threshold is not relevant. In the pricing of the firm conditional on $\tau_y > 2$ investors use $E(y|x)$ which equals $\beta_1 x$.

(ii) If $\tau_y = 2$ investors know that $y < h_2(x,2)$ and also that $y < x$. We need to distinguish between the binding case and the non-binding case. In the non-binding case, where $h_2(x,2) \leq x$, investors know that $y < h_2(x,2)$, so conditional on $\tau_y = 2$ investors set their beliefs as if the manager follows a disclosure threshold of $h_2(x,2)$. In the binding case, where $h_2(x,2) > x$, investors know that $y < x$, so it is equivalent to a disclosure threshold of x.

(iii) If $\tau_y = 1$ investors know that $y < x^*$ (where $x^* \leq x$) and also $y < h_2(x,2)$. Here again we should distinguish between a non-binding case, in which $h_2(x,2) < x^*$ (if such case exists), and a binding case in which $h_2(x,2) > x^*$. In the non-binding case the disclosure threshold is $h_2(x,2)$. In the binding case the disclosure threshold is x^*, which is independent of x.

The next Lemma provides an upper and lower bound for $\frac{\partial}{\partial x} h_2(x,2)$ and since the proof uses generic disclosure thresholds for each of the three scenarios above, it applies also to $\frac{\partial}{\partial x} h_2(x,1)$.

Lemma 12 *For any $p < 0.77$*

$$\frac{\partial}{\partial x} h_2(x, 2), \frac{\partial}{\partial x} h_2(x, 1) \in (2\beta_1 - 1, 2\beta_1).$$

Proof of Lemma 12.

In this proof we use a slightly different notation, as part of the proof is more general than our setting. Note that the first part of this proof is quite similar to the proof of Lemma 7.

Suppose that x and y have joint normal distribution and the agent is informed about y with probability p and uninformed with probability $1 - p$.[23] Conditional on being informed the agent's disclosure strategy is assumed to be as follows: with probability λ_i, $i \in \{1, .., K\}$, he discloses if his type is above $z_i(x)$, where the various $z_i(x)$ are determined exogenously such that $z_i(x) \leq h(x, \{z_i(x)\})$ for all i (which always holds in our setting). Note that $\sum_{i=1}^{K} \lambda_i = p$. Let's denote the conditional expectation of y given x and given the disclosure thresholds, $z_i(x)$, by $h(x, \{z_i(x)\})$.

By applying Bayes rule, $h(x, \{z_i(x)\})$ is given by:

$$h(x, \{z_i(x)\}) = \frac{(1-p) E[y|x] + \sum_{i=1}^{K} \lambda_i \int_{-\infty}^{z_i(x)} y\phi(y|x)\, dy}{(1-p) + \sum_{i=1}^{K} \lambda_i \Phi(z_i(x)|x)}.$$

Taking the derivative of $h(x, \{z_i(x)\})$ with respect to x and applying some algebraic manipulation (recall that $\frac{\partial E[y|x]}{\partial x} = \beta_1$) yields:

$$\frac{d}{dx} h(x, \{z_i(x)\}) = \beta_1 + \frac{\sum_{i=1}^{K} \lambda_i (z_i'(x) - \beta_1) \phi(z_i(x)|x)(z_i(x) - h(x, \{z_i(x)\}))}{(1-p) + \sum_{i=1}^{K} \lambda_i \Phi(z_i(x)|x)}. \tag{8}$$

We start by proving the supremum of $\frac{d}{dx} h(x, \{z_i(x)\})$.

Given that $z_i'(x) \geq 0$ and $(z_i(x) - h(x, \{z_i(x)\})) \leq 0$ for all $i \in \{1, .., K\}$ we have

$$\begin{aligned} \frac{d}{dx} h(x, \{z_i(x)\}) &\leq \beta_1 + \frac{\beta_1 \sum_{i=1}^{K} \lambda_i \phi(z_i(x)|x)(h(x, \{z_i(x)\}) - z_i(x))}{(1-p) + \sum_{i=1}^{K} \lambda_i \Phi(z_i(x)|x)} \\ &\leq \beta_1 + \max_{\substack{z_i \leq h(x) \\ i \in \{1,...,K\}}} \frac{\beta_1 \sum_{i=1}^{K} \lambda_i \phi(z_i|x)(h(x, \{z_i(x)\}) - z_i)}{(1-p) + \sum_{i=1}^{K} \lambda_i \Phi(z_i|x)}. \end{aligned}$$

Due to symmetry, for all $i \in \{1, .., K\}$ the maximum is achieved at some $z_i(x) = \hat{z}(x)$, which

[23] We apologize for the abuse of notation: the p in this proof corresponds to $p + p(1-p)$ in our model since this is the probability that the agent is informed about signal Y in the beginning of period 2.

is the same for all i. To see this, note that the FOC of the maximization with respect to $z_i(x)$ is

$$0 = \left(\phi'(z_i(x)|x)(h(x,\{z_i(x)\}) - z_i(x)) - \phi(z_i(x)|x)\right)\left((1-p) + \sum_{i=1}^{K} \lambda_i \Phi(z_i(x)|x)\right)$$
$$- \left(\sum_{i=1}^{K} \lambda_i \phi(z_i(x)|x)(h(x,\{z_i(x)\}) - z_i(x))\right)\phi(z_i(x)|x).$$

Since $\phi'(z_i(x)|x) = -\alpha(z_i(x) - \beta_1 x)\phi(z_i(x)|x)$ (for some constant $\alpha > 0$), this simplifies to

$$-\alpha(z_i(x) - \beta_1 x)(h(x,\{z_i(x)\}) - z_i(x)) = \frac{\sum_{i=1}^{K} \lambda_i \phi(z_i(x)|x)(h(x,\{z_i(x)\}) - z_i(x))}{(1-p) + \sum_{i=1}^{K} \lambda_i \Phi(z_i(x)|x)} + 1.$$

In the range $z_i(x) \leq h(x,\{z_i(x)\}) \leq \beta_1 x$, the LHS is decreasing in $z_i(x)$.[24] The RHS is the same for all i. Therefore, the unique solution to this system of FOC is for all $z_i(x)$ to be equal (and note that the maximum is achieved at an interior point since at $z_i(x) = h(x,\{z_i(x)\})$ the LHS is zero and the RHS is positive; and as $z_i(x)$ goes to $-\infty$ the LHS goes to $+\infty$ while the RHS is bounded).

Let $\hat{z}(x)$ be the maximizing value. Then

$$\begin{aligned}\frac{d}{dx}h(x,\{z_i(x)\}) &\leq \beta_1 + \frac{\beta_1 \sum_{i=1}^{K} \lambda_i \phi(\hat{z}(x)|x)(h(x,\{z_i(x)\}) - \hat{z}(x))}{(1-p) + p\Phi(\hat{z}(x)|x)} \\ &= \beta_1 + \frac{p\beta_1 \phi(\hat{z}(x)|x)(h(x,\{z_i(x)\}) - \hat{z}(x))}{(1-p) + p\Phi(\hat{z}(x)|x)}.\end{aligned}$$

The right hand side of the above inequality is identical to the slope in a Dye setting with exogenous disclosure threshold with probability of being uninformed $(1-p)$ and a disclosure threshold $\hat{z}(x)$, constant in x (see the discussion in Section 7.1). In such a setting, we can think of the effect of a marginal increase in x as the sum of two effects. The first effect is a shift by β_1 in both the distribution and the disclosure threshold. This will increase $h(x)$ by β_1. The second effect is a decrease in the disclosure threshold by β_1 (as the disclosure threshold does not change in x). Since $\hat{z}(x) < \beta_1 x$ we are in the decreasing part of the beliefs about y given no disclosure (to the left of the minimum beliefs). Therefore, the decrease in the disclosure threshold increases the beliefs about y by the change in the disclosure threshold times the slope of the beliefs about y given no disclosure. Since for $p < 0.95$ the slope of the beliefs about y given no disclosure is greater than -1, the latter effect increases the beliefs about y by less than β_1. The overall effect is therefore smaller than $2\beta_1$.

Next we prove the infimum of $\frac{d}{dx}h(x,\{z_i(x)\})$.

[24] Since $z_i(x) \leq h(x,\{z_i(\cdot)\})$ also $h(x,\{z_i(\cdot)\}) \leq E[x|y] = \beta_1 x$.

Equation (8) captures a general case with any number of potential disclosure strategies. In our particular case $K = 1$ where $i = 1$ represents the case of $\tau_y = 1$ and $i = 2$ represents the case of $\tau_y = 2$. So, in our setting equation (8) can be written as

$$\frac{d}{dx}h\left(x,\{z_i(x)\}\right) = \beta_1 + \frac{\lambda_1\left(z_1'\left(x\right)-\beta_1\right)\phi\left(z_1\left(x\right)|x\right)\left(z_1\left(x\right)-h\left(x,\{z_i(x)\}\right)\right)}{\left(1-p\right)+\sum_{i=1}^{2}\lambda_i\Phi\left(z_i\left(x\right)|x\right)} + \frac{\lambda_2\left(z_2'\left(x\right)-\beta_1\right)\phi\left(z_2\left(x\right)|x\right)\left(z_2\left(x\right)-h\left(x,\{z_i(x)\}\right)\right)}{\left(1-p\right)+\sum_{i=1}^{2}\lambda_i\Phi\left(z_i\left(x\right)|x\right)}.$$

When calculating $h_2\left(x,2\right)$ and $h_1\left(x,1\right)$ in our setting, the disclosure threshold, $z_i\left(x\right)$, in any possible scenario (the binding and non-binding case for both $\tau_y = 1$ and $\tau_y = 2$) takes one of the following three values: $h_i\left(x,\cdot\right)$, x or x^*. Note that whenever $z_i\left(x\right) = h\left(x,\{z_i(x)\}\right)$ we have $\frac{\left(z_i'(x)-\beta_1\right)\phi(z_i(x)|x)(z_i(x)-h(x,\{z_i(x)\}))}{(1-p)+\sum_{i=1}^{K}\lambda_i\Phi(z_i(x)|x)} = 0$.

For the remaining two cases ($z_i\left(x\right) = x$ and $z_i\left(x\right) = x^*$), for all $i \in \{1,2\}$ we have $z_i'\left(x\right) \leq 1$ and $\left(z_i\left(x\right) - h\left(x,\{z_i(x)\}\right)\right) \leq 0$. This implies

$$\frac{d}{dx}h\left(x,\{z_i(x)\}\right) \geq \beta_1 - \frac{\left(1-\beta_1\right)\sum_{i=1}^{K}\lambda_i\phi\left(z_i\left(x\right)|x\right)\left(h\left(x,\{z_i(x)\}\right)-z_i\left(x\right)\right)}{\left(1-p\right)+\sum_{i=1}^{K}\lambda_i\Phi\left(z_i\left(x\right)|x\right)}.$$

Using the same symmetry argument for the first order condition as before, $\frac{d}{dx}h\left(x,\{z_i(x)\}\right)$ is minimized for some $z^{\min}\left(x\right)$, and hence,

$$\frac{d}{dx}h\left(x,\{z_i(x)\}\right) \geq \beta_1 + \frac{p\left(1-\beta_1\right)\phi\left(z^{\min}\left(x\right)|x\right)\left(h\left(x,\{z_i(x)\}\right)-z^{\min}\left(x\right)\right)}{\left(1-p\right)+p\Phi\left(z^{\min}\left(x\right)|x\right)}.$$

The right hand side of the above inequality is identical to the slope in a Dye setting with exogenous disclosure threshold in which: the probability of being uninformed is $(1-p)$, the threshold is $z^{\min}\left(x\right)$, and $\frac{\partial}{\partial x}z^{\min}\left(x\right) = 1$ (see the discussion in Section 7.1). In such a setting, we can think of the effect of a marginal increase in x as the sum of two effects. The first is a shift by β_1 in both the distribution and the disclosure threshold. This will increase beliefs about y by β_1. The second effect is an increase in the disclosure threshold by $(1-\beta_1)$ (as the disclosure threshold increases by 1). Since $z^{\min}\left(x\right) < \beta_1 x$ we are in the decreasing part of the beliefs about y given no disclosure (to the left of the minimum beliefs). Therefore, the increase in the disclosure threshold decreases the beliefs about y by the change in the disclosure threshold, $(1-\beta_1)$, times the slope of the beliefs about y given no disclosure. Since for $p < 0.95$ the slope of the beliefs about y given no disclosure is greater than -1, the latter effect decreases the beliefs about y by less than $(1-\beta_1)$. The overall effect is therefore greater than $\beta_1 - (1-\beta_1) = 2\beta_1 - 1$.

The reasoning we have presented is independent of the actual thresholds, so the bounds apply to $h_2(x,1)$ as well. ■

This covers the range $h_2(x,2) \geq x^*$.

For the case $h_2(x,2) < x^*$ (if such case exists) we claim that $\frac{\partial}{\partial x} h_2(x,2) = \beta_1$.

To see this, note that $h_2(x,2)$ is a weighted average of the beliefs about y over the three scenarios $\tau_y = 1$, $\tau_y = 2$ and $\tau_y > 2$. That is, we can write

$$h_2(x,2) = \lambda_1 g_1 + \lambda_2 g_2 + (1 - \lambda_1 - \lambda_2) g_3,$$

where $\lambda_i = \Pr(\tau_y = i | ND_y)$ and $g_i = E(y | \tau_y = i, ND_y)$ for $i = 1,2,3$ (where $i = 3$ represents the case of $\tau_y > 2$). ND_y stands for No-Disclosure of y (where x was disclosed at $t = 2$). Since $h_2(x,2) < x^*$ the disclosure threshold for both $\tau_y = 2$ and $\tau_y = 1$ is $h_2(x,2)$.

Assume, by contradiction, that $\frac{\partial}{\partial x} h_2(x,2) > \beta_1$. Then, an increase in x increases $h_2(x,2)$ by more than the increase in the expectation of y (which is β_1) and therefore, the probability of obtaining a signal below the disclosure threshold increases for both the first and the second period. This implies that both λ_1 and λ_2 increase. In addition, note that the increase in g_1 and in g_2 is lower than $\frac{\partial}{\partial x} h_2(x,2)$ and the increase in g_3 is β_1 - which is also lower than $\frac{\partial}{\partial x} h_2(x,2)$. The fact that both g_1 and g_2 are lower than g_3 leads to a contradiction, since an increase in x puts more weight on the lower values (λ_1 and λ_2 increase) and in addition all the values g_1, g_2, g_3 increase at a rate weakly lower than the assumed increase in $h_2(x,2)$. A symmetric argument can be made when assuming by contradiction that $\frac{\partial}{\partial x} h_2(x,2) < \beta_1$. The case of $\frac{\partial}{\partial x} h_2(x,2) = \beta_1$ does not lead to a contradiction, as an increase in x does not affect the probabilities λ_1, λ_2 and the derivatives of g_1 and g_2 and g_3 are all equal to β_1.

Finally, we analyze $h_2(x,1)$.

Recall that Lemma 12 applies also to $h_2(x,1)$. However, for $h_2(x,1)$ we can show tighter bounds.

1) If $h_2(x,1) < x$ (the non-binding case) then when pricing the firm at $t = 2$ investors know that if the agent learned y (at either $t = 1$ or $t = 2$) then $y < h_2(x,1)$. If the agent did not learn y then investors use in their pricing $E(Y|x) = \beta_1 x$. So, the beliefs about y are a weighted average of $E(Y|y < h_2(x,1))$ and $E(Y|x) = \beta_1 x$. This is similar to a Dye (1985) and Jung and Kwon (1988) setting, and therefore, in equilibrium we have $\frac{\partial}{\partial x} h_2(x,1) = \beta_1$.

2) Next, we show that for x such that $h_2(x,1) > x$ (if such case exists) $\frac{\partial}{\partial x} h_2(x,1) \in (2\beta_1 - 1, \beta_1)$.

The argument is similar to the one we made in the proof of Lemma 11 (that $\frac{\partial}{\partial x} h_1(x,1) \in (2\beta_1 - 1, \beta_1)$, for x such that $h_1(x,1) > x$). First note that for $h_2(x,1) > x$ investors' beliefs about y conditional on that the agent has learned y are independent of whether he learned y at $t = 1$ or at $t = 2$. Moreover, given that $\tau_y \leq 2$ investors know that $y < x$. So, from investors' perspective, it doesn't matter if the agent learned y at $t = 1$ or at $t = 2$. Their pricing, $h_2(x,1)$, will reflect a weighted average between $E(Y|y < x)$ and $E(Y|\tau_y > 2, x) = \beta_1 x$. From here on the proof is qualitatively the same as in the proof for $\frac{\partial}{\partial x} h_1(x,1) \in (2\beta_1 - 1, \beta_1)$, where the only quantitative difference is the probability that the agent learned y.

QED Claim 2

7.3.2 Lemma 8

Proof of Lemma 8. For simplicity of exposition, we partition the support of x into two cases: realizations of x for which $\beta_2(x + h_2(x,2)) \geq P_2(\emptyset)$ and for which $\beta_2(x + h_2(x,2)) < P_2(\emptyset)$.[25]

Case I - $\beta_2(x + h_2(x,2)) \geq P_2(\emptyset)$ (i.e. an agent that does not learn the second signal will prefer to disclose x at $t = 2$)

Rewriting $E(U|\tau_x = 1, \tau_y \neq 1, t_x = 1, x) - E(U|\tau_x = 1, \tau_y \neq 1, t_x \neq 1)$ yields:

$$\beta_2 [x + h_1(x,1) + h_2(x,1) - h_2(x,2)] - P_1(\emptyset)$$
$$+ p\beta_2 \left[\int_{h_2(x,1)}^{\infty} (y - h_2(x,1)) f(y|x) \, dy - \int_{h_2(x,2)}^{\infty} (y - h_2(x,2)) f(y|x) \, dy - \int_{y^H(x)}^{\infty} (h_2(y,2) - x) f(y|x) \, dy \right].$$

The derivative of this expression with respect to x has the same sign as

$$D = 1 + \frac{\partial}{\partial x} (h_1(x,1) + h_2(x,1) - h_2(x,2)) + p[A + B + C], \tag{9}$$

where

$$A = \frac{\partial}{\partial x} \int_{h_2(x,1)}^{\infty} (y - h_2(x,1)) f(y|x) \, dy$$
$$B = -\frac{\partial}{\partial x} \int_{h_2(x,2)}^{\infty} (y - h_2(x,2)) f(y|x) \, dy$$
$$C = -\frac{\partial}{\partial x} \int_{y^H(x)}^{\infty} (h_2(y,2) - x) f(y|x) \, dy.$$

[25] Note that on the equilibrium path we are always in case I, i.e., $\beta_2(x + h_2(x,2)) \geq P_2(\emptyset)$.

To evaluate this derivative we use the following, easy to obtain, equations:

$$\frac{\partial}{\partial x} f(y|x) = -\beta_1 \frac{\partial}{\partial y} f(y|x),$$
$$\frac{\partial}{\partial x}(F(y(x)|x)) = f(y(x)|x)\left(\frac{\partial}{\partial x} y(x) - \beta_1\right).$$

Next, we analyze the three terms A, B, and C. Note that the derivative with respect to the limits of integrals for A, B and C is zero.

$$A = -\frac{\partial h_2(x,1)}{\partial x}(1 - F(h_2(x,1)|x)) - \beta_1 \int_{h_2(x,1)}^{\infty} (y - h_2(x,1)) \frac{\partial}{\partial y} f(y|x)\, dy.$$

Integrating by parts (w.r.t. y) the term $\int_{h_2(x,1)}^{\infty} (y - h_2(x,1)) \frac{\partial}{\partial y} f(y|x)\, dy$ yields:

$$\begin{aligned}
&\int_{h_2(x,1)}^{\infty} (y - h_2(x,1)) \frac{\partial}{\partial y} f(y|x)\, dy \\
&= -(h_2(x,1) - h_2(x,1)) f(h_2(x,1)|x) - \int_{h_2(x,1)}^{\infty} f(y|x)\, dy = -(1 - F(h_2(x,1)|x)).
\end{aligned}$$

Plugging it back to A we get

$$A = -\left(\frac{\partial h_2(x,1)}{\partial x} - \beta_1\right)(1 - F(h_2(x,1)|x)).$$

Next, we calculate B:

$$\begin{aligned}
B &= \int_{h_2(x,2)}^{\infty} \frac{h_2(x,2)}{\partial x} f(y|x)\, dy + \beta_1 \int_{h_2(x,2)}^{\infty} (y - h_2(x,2)) \frac{\partial}{\partial y} f(y|x)\, dy \\
&= \left(\frac{\partial h_2(x,2)}{\partial x} - \beta_1\right)(1 - F(h_2(x,2)|x)).
\end{aligned}$$

Finally,

$$\begin{aligned}
C &= \left(1 - F\left(y^H(x)|x\right)\right) + \beta_1 \int_{y^H(x)}^{\infty} (h_2(y,2) - x) \frac{\partial}{\partial y} f(y|x)\, dy \\
&= \left(1 - F\left(y^H(x)|x\right)\right) - \beta_1 \int_{y^H(x)}^{\infty} \frac{\partial h_2(y,2)}{\partial y} f(y|x)\, dy.
\end{aligned}$$

Substituting A, B and C back to (9) and re-arranging terms yields:

$$\begin{aligned}
D &= 1 + \frac{\partial}{\partial x}(h_1(x,1) + h_2(x,1) - h_2(x,2)) \\
&\quad - p \left[\begin{array}{c} \left(\frac{\partial h_2(x,1)}{\partial x} - \beta_1\right)(1 - F(h_2(x,1)|x)) + \left(\frac{\partial h_2(x,2)}{\partial x} - \beta_1\right)(1 - F(h_2(x,2)|x)) + \\ \left(1 - F\left(y^H(x)|x\right)\right) - \beta_1 \int_{y^H(x)}^{\infty} \frac{\partial h_2(y,2)}{\partial y} f(y|x)\, dy \end{array} \right] \\
&= (1-p)\left(1 + \frac{\partial}{\partial x}(h_1(x,1) + h_2(x,1) - h_2(x,2))\right) \\
&\quad + p \left[\begin{array}{c} 1 + \frac{\partial h_1(x,1)}{\partial x} + \frac{\partial h_2(x,1)}{\partial x} F(h_2(x,1)|x) - F(h_2(x,1)|x)\beta_1 - \frac{\partial h_2(x,2)}{\partial x} F(h_2(x,2)|x) \\ +F(h_2(x,2)|x)\beta_1 + \left(1 - F\left(y^H(x)|x\right)\right) - \beta_1 \int_{y^H(x)}^{\infty} \frac{\partial h_2(y,2)}{\partial y} f(y|x)\, dy \end{array} \right]
\end{aligned}$$

Additional rearranging yields:

$$
\begin{aligned}
D &= (1-p)\left(1+\frac{\partial}{\partial x}\left(h_1(x,1)+h_2(x,1)-h_2(x,2)\right)\right) \\
&+ p\left[1+\frac{\partial h_1(x,1)}{\partial x}+\left(\frac{\partial h_2(x,1)}{\partial x}-\beta_1\right)F\left(h_2(x,1)\,|x\right)-\left(\frac{\partial h_2(x,2)}{\partial x}-\beta_1\right)F\left(h_2(x,2)\,|x\right)\right] \\
&+ p\beta_1\int_{y^H(x)}^{\infty}\frac{1}{\beta_1}-\frac{\partial h_2(y,2)}{\partial y}f(y|x)\,dy.
\end{aligned}
$$

Since $\frac{\partial h_2(x,1)}{\partial x}\le\beta_1$ (see Claim 2) and $F\left(h_2(x,2)\,|x\right)\ge F\left(h_2(x,1)\,|x\right)$ we have

$$
\begin{aligned}
D &\ge (1-p)\left(1+\frac{\partial}{\partial x}\left(h_1(x,1)+h_2(x,1)-h_2(x,2)\right)\right) \\
&+ p\left[1+\frac{\partial h_1(x,1)}{\partial x}+\left(\frac{\partial h_2(x,1)}{\partial x}-\frac{\partial h_2(x,2)}{\partial x}\right)F\left(h_2(x,2)\,|x\right)\right] \\
&+ p\beta_1\int_{y^H(x)}^{\infty}\frac{1}{\beta_1}-\frac{\partial h_2(y,2)}{\partial y}f(y|x)\,dy \\
&= \left(1-p\left(1-F\left(h_2(x,2)\,|x\right)\right)\right)\left(1+\frac{\partial}{\partial x}\left(h_1(x,1)+h_2(x,1)-h_2(x,2)\right)\right)+ \\
&+ p\left(1-F\left(h_2(x,2)\,|x\right)\right)\left(1+\frac{\partial h_1(x,1)}{\partial x}\right)+p\beta_1\int_{y^H(x)}^{\infty}\frac{1}{\beta_1}-\frac{\partial h_2(y,2)}{\partial y}f(y|x)\,dy \\
&= \left(1-p\left(1-F\left(h_2(x,2)\,|x\right)\right)\right)\left(1+\frac{\partial}{\partial x}\left(h_1(x,1)+h_2(x,1)-h_2(x,2)\right)\right)+ \\
&+ p\int_{h_2(x,2)}^{y^H(x)}\left(1+\frac{\partial h_1(x,1)}{\partial x}\right)f(y|x)\,dy+p\int_{y^H(x)}^{\infty}2+\frac{\partial h_1(x,1)}{\partial x}-\beta_1\frac{\partial h_2(y,2)}{\partial y}f(y|x)\,dy \\
&\ge \left(1-p\left(1-F\left(h_2(x,2)\,|x\right)\right)\right)\left(1+\frac{\partial}{\partial x}\left(h_1(x,1)+h_2(x,1)-h_2(x,2)\right)\right) \\
&+ p\int_{y^H(x)}^{\infty}2+\frac{\partial h_1(x,1)}{\partial x}-\beta_1\frac{\partial h_2(y,2)}{\partial y}f(y|x)\,dy
\end{aligned}
$$

So, the following two conditions are sufficient for the proof of Case I.

For all x:

1. $\frac{\partial}{\partial x}h_1(x,1)+\frac{\partial}{\partial x}h_2(x,1)\ge\frac{\partial}{\partial x}h_2(x,2)-1.$

2. $\frac{\partial h_2(y,2)}{\partial y}\le\left(2+\frac{\partial h_1(x,1)}{\partial x}\right)\frac{1}{\beta_1}$ for any $y>x$.

Case II - $\beta_2\left(x+h_2(x,2)\right)<P_2(\emptyset)$ (i.e. an agent that does not learn the second signal will prefer to not disclose x at $t=2$)

The analysis of Case I was for generic bounds of the integrals $h_2(x,1)$ and $y^H(x)$. The difference between Case I and Case II is that the price at $t=2$ given no disclosure of y (which occurs when the agent does not obtain a signal y or obtains a low realization of y) is $P_2(\emptyset)$ in Case II and $\beta_2(x + h_2(x, 2))$ in Case I. Therefore, the expected payoff of the agent in Case II is less sensitive to x than in Case I. As a result, the fact that for values of x for which $\beta_2(x + h_2(x, 2)) \geq P_2(\emptyset)$ (in Case I) $\frac{\partial}{\partial x}(E(U|\tau_x = 1, \tau_y \neq 1, t_x = 1) - E(U|\tau_x = 1, \tau_y \neq 1, t_x \neq 1)) > 0$ implies that it also holds for $\beta_2(x + h_2(x, 2)) < P_2(\emptyset)$.

To summarize the analysis of Partially Informed Agents, conditions 1 and 2 above are sufficient for both Case I and Case II. Claim 2 established that condition 2 above holds.

So, it is only left to show that also condition 1 holds. For any $\beta_1 > \frac{1}{2}$, it is immediate to see that condition 1 holds since the LHS of condition 1 is greater than $2(2\beta_1 - 1) > 0$ and the RHS is less than $2\beta_1 - 1$ (again by Claim 2) For the case $\beta_1 < \frac{1}{2}$ we use the assumption that x^* satisfies $h_2(x^*, 1) \leq x^*$. For such x^* we know from Claim 2 that $\frac{\partial}{\partial x} h_2(x, 1) = \beta_1$ for all $x \geq x^*$. Substituting this into condition 1 above yields $\frac{\partial}{\partial x} h_1(x, 1) + \beta_1 \geq \frac{\partial}{\partial x} h_2(x, 2) - 1$ which given Claim 2 is always satisfied. ■

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